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Modeling the hydraulic washing-out process of amber-bearing rocks during amber extraction

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Abstract

Purpose. The study focuses on the hydromechanical extraction of amber from sandy amber-bearing deposits in the Volyn Polissia region to identify conditions for efficient washing-out and optimize the technological parameters of extraction.

Methods. The research employs a modeling approach for hydraulic washing-out based on the conditions of the Klesiv deposit. A mathematical model was developed to simulate the motion of solid particles in a turbulent flow, considering the stochastic nature of flow velocity and particle size distribution. The model integrates a system of differential equations describing particle movement and a block diagram of a statistical algorithm for evaluating the probability of amber particle detachment. Laboratory experiments were conducted using samples collected from productive sand horizons of the deposit.

Findings. It was established that the probability of amber particle detachment increases significantly with higher near-bed velocity dispersion and granulometric heterogeneity of the rocks. Numerical simulations demonstrated that fine particles ($d \leq 1.4$ mm) detach without rolling, while larger ones predominantly roll along the surface. A quantitative relationship was obtained between detachment probability and environmental parameters, allowing for a predictive assessment of hydraulic washing-out efficiency.

Originality. A novel mathematical model was proposed, which accounts for the interaction between turbulent stochastic factors and the physical properties of particles. A statistical approach was applied for the first time to determine the critical conditions for amber particle detachment from the rock matrix.

Practical implications. The results can be used to improve existing and develop new hydromechanical technologies for amber extraction, particularly in challenging sandy-clayey formations with high heterogeneity. Determining the critical flow parameters that ensure the highest probability of amber particle detachment enables the optimization of hydraulic mining giant operation modes and the designing of efficient slurry transport systems.

Keywords: *amber, hydraulic washing-out, amber-bearing rocks, hydromechanical method, turbulent flow, slurry*

1. Introduction

Ukraine possesses substantial reserves of valuable amber. The primary amber deposits are concentrated in the Polissia region, specifically within the Pripyat amber-bearing basin, which spans the northern parts of the Rivne, Volyn, Zhytomyr, and Kyiv regions. Industrial-scale extraction is carried out at deposits located in the Sarny district (Vilne, Klesiv) and the Volodymyrets district of the Rivne region [1]-[3].

Approximately 6% of the world's amber reserves are located in the Rivne region, with total estimated reserves reaching 100 thousand tonnes. The amber is predominantly embedded in sandy and sandy-clayey soils at depths of up to 15 meters, making these conditions suitable for research and implementing new extraction technologies [4], [5].

The modern amber-bearing landscapes of Volyn Polissia are bounded to the north by the state border of Ukraine with Belarus; to the east, by outcrops of the crystalline basement rocks of the Ukrainian Shield; to the south, by the rise of the Volyn loess plateau; and to the west, by the channel of the Western Bug River. The total area of the amber-bearing region is approximately 35 thousand square kilometers [6], [7].

A distinguishing feature of Volyn Polissia is its extensive forest coverage. The forests primarily comprise pine-oak and oak-hornbeam associations, while shrublands occupy about 45% of the territory. Floodplains of bog-type landscapes cover roughly 15% of the area [8].

The soil cover of the region is heterogeneous, with more than 35% of the area occupied by sod-podzolic soils. Common soil types include gleyed sod soils and peat-bog varieties. The parent materials are predominantly clay-sandy and sandy deposits. Weakly and moderately podzolized sod-podzolic sandy soils cover most of the territory. Loamy and sod-podzolic sandy loam soils occupy significantly smaller areas and are mainly found in the southern part of the region. Bog soils are concentrated primarily in lowlands, floodplains, and river valleys. Half of all wetlands in the area are peatlands [9], [10].

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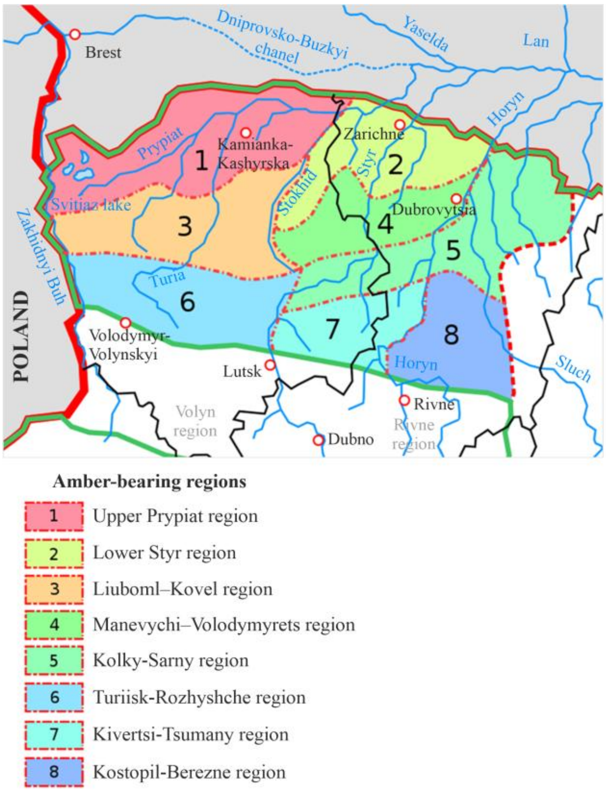

Figure 1. Zoning of the Volyn Polissia region

According to research findings [11], three zones and four districts with industrial concentrations of amber have been identified in the Rivne-Volyn region. The total area of amber-bearing horizons in the Rivne region is estimated at 3.810 km², representing about 18% of its territory.

As of January 1, 2024, four deposits were registered in the national mineral reserve balance: Klesivske, Volodymyrets-Skhidnyi, Zolote (Southeastern area), and Vilne. The classified reserves of these deposits are 63.61 tonnes under Category C1 and 162.87 tonnes under Category C2. Amber-bearing sands are of variable grain size, predominantly fine to medium-grained, gray, dark gray, or greenish-gray in color, sometimes containing up to 3-5% silica and fragments of crystalline rocks. Amber fragments are of various shapes and sizes, with oxidation crusts up to 1-2 mm thick. Most pieces measure 1-2 cm, with occasional fragments reaching 5-10 cm.

Amber is extracted from sandy amber-bearing deposits using two principal methods: mechanical and hydromechanical [12], [13]. The mechanical method involves the excavation of open pits or underground masses using mechanical means. It includes uncovering productive soil layers, drilling, transportation of overburden, screening, washing of the ore, and land reclamation. This method employs a complex of machines and is currently considered outdated. The hydromechanical method utilizes high-pressure water jets to disintegrate amber-bearing rocks and hydraulic flows to transport amber to the deposit's surface.

Geological structure, types of amber-bearing horizons, and spatial variability of the thickness of productive layers within the Pripyat amber-bearing basin have been examined in several in-depth studies [14], [15]. Particular emphasis has been placed on the importance of detailed stratigraphic subdivision of the sandy-clayey sequences, within which the highest concentrations of amber are typically found. The works [16]-[18] also highlight the patterns of amber deposition in alluvial, diluvial, and peat-forming deposits developed within river valleys and on the slopes of ancient terraces.

Recent literature has also devoted significant attention to analyzing amber extraction methods, their efficiency, and their environmental impact [19]-[21]. Scholars [22]-[25] compare conventional mechanical, hydromechanical, and combined extraction techniques, emphasizing the urgent need to implement resource-efficient technologies tailored to the hydrogeological conditions of specific deposits. According to [26]-[30], one of the key directions in extraction process optimization involves mathematical modeling of hydrodynamic processes during hydraulic washing-out. Such modeling enables the prediction of extraction efficiency and reduction of anthropogenic impact on the environment.

In certain industrial zones, a mechano-hydraulic extraction method is applied. This approach involves mobilizing the amber-bearing material in situ using a mechanical working unit and then transporting the resulting amber-containing slurry to the surface via vertical mine workings.

Research has established that at the Klesiv deposit in the Rivne region, amber occurs at depths of up to 15 meters exclusively within sandy soils. This enables efficient extraction using various established methods with minimal operational costs. Mineralogical analysis of amber-bearing sands at the deposit [31], [32] reveals a significant amber content (on average 50-120 g/m³), which justifies the intensification of extraction efforts and the need to determine the specific environmental conditions under which maximum recovery is achieved. Experimental investigations carried out in amber-bearing sand masses have also identified other valuable chemical compounds, the most notable of which is aluminum oxide (alumina, 24.59%) – a key raw material in aluminum production [33]. This adds economic value to the deposits and supports the potential expansion of raw material processing, including alumina recovery.

Given the relevance of ensuring efficient and environmentally sustainable amber extraction under conditions of heterogeneous sandy (media particularly at deposits such as Klesiv), there is a clear need for a scientifically grounded approach to modeling the hydraulic washing-out process. This study aims to develop a simulation model for the interaction between turbulent flow and solid particles within amber-bearing rock, to determine the conditions for probable detachment of amber particles, and to evaluate the influence of granulometric characteristics and flow parameters on the efficiency of hydromechanical extraction.

2. Methodology

2.1. Initial conditions and modeling approaches for the hydraulic washing-out process

The primary investigations were conducted at the Fedorivske occurrence, located south of the Klesiv deposit in the Sarny district. The productive layers at this site are composed of sand. Amber fragments found within the occurrence range in size from 0.3 cm to 5-15 cm. The thickness of the productive strata varies from 0.5 to 16.0 meters, while amber content ranges from 1 to 420 g/m^3 [34]-[36].

During modeling, the formation of unloading surfaces under complex stress-strain conditions in amber-bearing rock masses was considered a mechanism for controlling the behavior of geotechnical engineering systems [37]-[40]. This was achieved by artificially structuring localized control zones within the technically loaded rock mass. Analytical solutions for forming unloading surfaces are based on the motion of solid particles in a hydraulic flow, independent of their attachment to the rock matrix.

Experimental studies were carried out in the university's laboratory using samples of amber-bearing sands collected from a representative site at five different locations and various depths within the industrially exploited Klesiv deposit, operated by KKNK "Tekhnobud".

The laboratory modeling data were used to predict localized erosion caused by spiral flows, circulating currents, separated jets, and other forms of heterogeneous flow activity. However, precise physical modeling of washing-out processes is inherently limited due to the stochastic nature of many influencing factors, such as hydraulic resistance of the channel driven by viscosity at low Reynolds numbers, inaccuracies in reproducing experimental conditions, and the inability to simultaneously satisfy a large number of similarity criteria under identical conditions. Some of the most common reasons for these limitations are outlined below. Since the washing-out process involves the detachment of individual particles, the modeling typically focuses on the force balance acting on a single particle located on the bottom or slope of the eroded surface.

In the development of the simulation model, the following tasks were addressed:

1. The model accounts for variations in vertical, horizontal, and shear forces acting on a particle, corresponding to the characteristics of the kinematic structure of natural flows responsible for the washing-out process. The random nature of flow velocity fluctuations above the particle is considered. In particular, sequences of random numbers are simulated with the same time-averaged values, variances, and autocorrelation functions as those observed in the original sequence of force fluctuations.

2. Random values for particle diameters and roughness element heights are generated. This simulates realistic conditions of particle resting positions about neighboring grains and the actual granulometric composition of bottom sediments.

3. The detachment time is determined for each particle in a sufficiently large sample set (containing particles of various diameters) from the complete system of differential equations governing particle motion. The probabilities of particle entrainment are then calculated, taking into account the duration of flow action on each particle. A particle is considered detached from the bed if the detachment time is less than or equal to the duration of flow influence.

2.2. Mathematical modeling of particle motion during hydromechanical amber extraction

Consider a spherical particle resting on the bed surface and subjected to a turbulent flow (Fig. 2).

It is also possible to consider particles of more complex shapes, such as ellipsoidal, cubic, or other configurations. Naturally, the hydrodynamic pressure acting on the particle surface will be distributed non-uniformly (Fig. 3). The resultant normal pressure acting on the surface may be represented by the force $P = \int P(S)dS$, where S – denotes the surface area of the spherical particle.

In addition to normal pressure, tangential (shear) stresses of viscous origin arise on the particle's surface. The resultant of these stresses is denoted by T (Fig. 4).

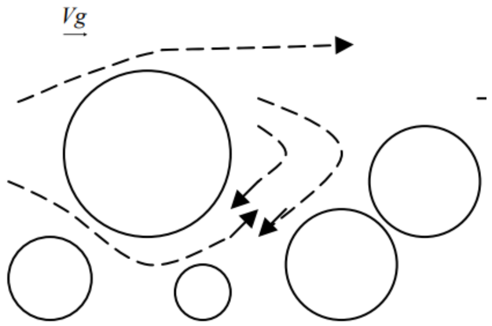

Figure 2. Flow of a turbulent stream around a spherical particle

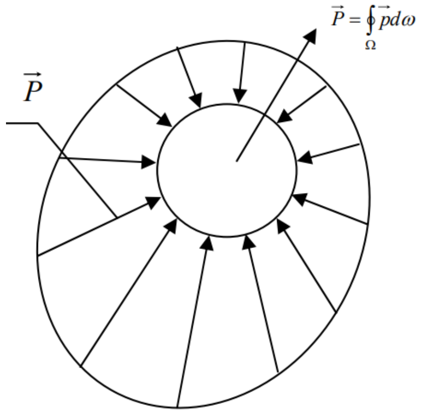

Figure 3. Hydrodynamic pressure acting on the surface of a particle

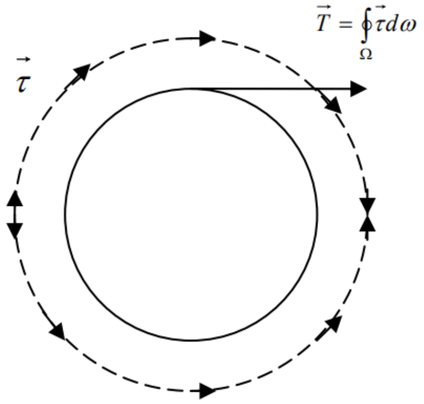

Figure 4. Generation of shear stresses on the surface of a spherical particle

A stationary coordinate system is chosen, and its origin is at the point of contact O.

Figure 5 illustrates a representative scheme of the forces acting on a particle in a turbulent flow: G – gravitational force; F_{Arch} – Archimedes (buoyant) force; $P = \{P_x, P_y\}$ – force vector exerted by the turbulent flow on the particle; $R = \{Rx, Ry\}$ – reaction force acting on the particle from the surface; T – resultant shear force due to viscous stresses.

In modeling the hydraulic washing-out process of amber-bearing sands, it is assumed that at the initial moment, the particle is detached from the bed by an infinitesimally small distance, such that the bed's reaction force acting on the particle is zero.

Applying D'Alembert's principle wherein all forces acting on the particle, including inertial forces, are in mutual equilibrium, the motion of the spherical particle can be described by the following system of differential equations with corresponding initial conditions.

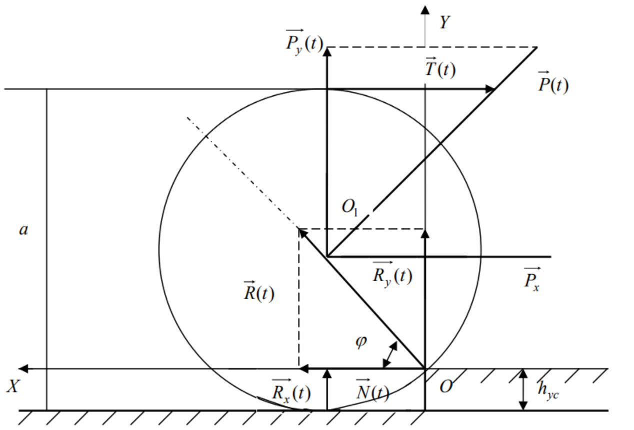

Figure 5. Force diagram acting on a particle in a turbulent flow

$$m\frac{d^2X_c}{dt^2}=F_x\left(t\right); \tag{1}$$

$$m\frac{d^2Y_c}{dt^2}=F_y\left(t\right); \tag{2}$$

$$X_c\left(X,Y,O\right)=X_0;\; Y_c\left(X,Y,O\right)=Y_0.$$

$$\frac{dX_c}{dt_{|t=0}}=0;\; \frac{dY_c}{dt_{|t=0}}=0; \tag{3}$$

$$\frac{d^2\varphi}{dt^2}=\frac{1}{J}\sum_{i=1}^{3}M_i\left(t\right); \tag{4}$$

$$\varphi\left(0\right)=\varphi_0;$$

$$\frac{dX\varphi_c}{dt_{|t=0}}=0, \tag{5}$$

where:

$X_c = X_c(X, Y, t)$, $Y_c = Y_c(X, Y, t)$ – coordinates of the particle's center of mass;

m – mass of the particle, accounting for the added mass of water;

t – time;

ϕ – angle of rotation;

J – moment of inertia of the particle;

X_0, Y_0 – coordinates of the particle's center of mass at the initial time;

ϕ_0 – the initial angle of rotation;

F_x, F_y – components of the resultant force projected onto the O_x and O_y axes, respectively;

M_i – moment of the acting forces concerning the center of rotation O.

Equations (1) and (2) represent the differential equations governing the translational motion of the particle's center of mass with initial conditions defined in (3), while Equation (4) expresses Newton's law for rotational motion of the particle with initial conditions specified in (5).

Three possible modes of particle detachment are considered. The first scenario corresponds to the moment when the sum of the vertical components of the active forces exceeds the sum of the particle's weight and the vertical component of friction. The detachment condition in this case is written as:

$$F_x=0;\; F_y\geq 0. \tag{6}$$

The second scenario involves the rotational motion of the particle around a point and detachment from the contact point when the normal reaction force becomes zero:

$$R=\sqrt{R_x^2+R_y^2}=0. \tag{7}$$

The third scenario involves the particle rolling over a barrier and swinging away. In this case, the detachment condition takes the form:

$$\varphi\left(t\right)>\varphi=\frac{\pi}{2}. \tag{8}$$

Equation (8) characterizes the motion of a solid particle in a hydraulic flow during the moment of detachment of the amber particle from the rock matrix.

Amber extraction is feasible using the hydromechanical method, in which the sandy massif becomes saturated with a water-enriched mixture and is subjected to mechanical vibrations induced by the working tool [41], [42].

The motion of amber within the sandy massif is governed by resistance similar to that of dry friction. When a particle is placed in a medium undergoing horizontal translational oscillations with frequency f and oscillation radius r, it experiences resistance akin to that caused by dry friction. This problem was previously analyzed by V.V. Gortinsky, H.Ye. Ptushkina, and I.I. Blekhman, who presented key results and their interpretation in the context of hydromechanical amber extraction.

To determine the ascent velocity of amber, the following system of differential equations is considered, which describes the motion of a particle relative to the oscillating medium:

$$m_1\ddot{x}=m_0\left(\Delta-1\right)rf^2\cos ft-F_h\frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}}; \tag{9}$$

$$m_1\ddot{y}=m_0\left(\Delta-1\right)rf^2\cos ft-F_h\frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}}; \tag{10}$$

$$m_1\ddot{z}=m_0\left(\Delta-1\right)g-F_v\frac{\dot{z}}{\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}}; \tag{11}$$

$$\Delta=\frac{\rho}{\rho_0}; \tag{12}$$

$$\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\neq 0, \tag{13}$$

where:

F_v – resistance force to the relative vertical displacement of the particle, N;

m_1 – mass of the particle, including the added mass of the surrounding medium, kg;

m_0 – mass of the medium in a volume equal to that of the particle, kg;

$\dot{x}$, $\dot{y}$, $\dot{z}$ – projections of the particle's relative velocity in the medium along the axes of a Cartesian coordinate system, m.

Based on these considerations, the following expression was proposed for determining the ascent velocity of amber:

$$v=\frac{a^2}{\sqrt{\left(\frac{F}{gm}\right)^2-a^2}}\cdot\frac{\alpha}{k_m}; \tag{14}$$

$$\alpha=\begin{cases}Af, & A^2f<kg\\ \dfrac{kg}{f}, & A^2f>kg\end{cases}; \tag{15}$$

$$a = \rho - 1; \tag{16}$$

$$\rho = \frac{\rho_c}{\rho_s}, \tag{17}$$

where:

v – ascent velocity of the amber particle, m/s;

m – mass of the amber particle, kg;

a – Archimedes parameter for the amber particle;

g – gravitational acceleration, m/s^2;

k_m – added mass coefficient;

f – frequency of oscillation of the working tool, Hz;

F – resistance force of the sandy layer to the relative motion of the amber particle;

ρ_s – density of the amber particle, kg/m^3;

ρ_c – density of the water-sand medium, kg/m^3;

A – amplitude of oscillation of the working tool, m;

k – coefficient of friction.

Equation (14) was derived for the case of a particle with lower density ascending through a layer of heavier material (a water-saturated soil medium) under oscillations of the working tool within the plane of the enclosing medium. In this scenario, apart from inertial forces, Archimedes' force, and gravity, the only additional force acting on the particle is the resistance to its relative motion. The particle's trajectory under such conditions typically follows a spiral path.

However, under the given conditions, there is limited information regarding the determination of the resistance force F. Considering its dimensional characteristics and its physical interpretation as the interaction between the amber particle and the particles of the water-saturated soil matrix, the following expression was proposed for estimating this force:

$$\frac{F}{gm} = \phi(1 + a); \tag{18}$$

where:

ϕ – resistance coefficient of the sandy medium to the motion of the amber particle.

Substituting Expression (18) into Equation (14), a final expression was obtained for the ascent velocity of an amber particle in a water-saturated soil medium:

$$\frac{k_m v}{\alpha} = \frac{a^2}{\sqrt{\phi^2 (1 + a)^2 - a^2}}. \tag{19}$$

The resulting Expression (19) accounts for the influence of the density difference between the host rock particles and the valuable mineral, as well as the working tool's amplitude and frequency of oscillations. The effect of gas bubbles is considered only indirectly through modification of the density of the amber-bearing medium.

To determine the value of the coefficient ϕ, we rely on experimental results obtained during the ascent of amber particles under laboratory and field conditions [43], [44]. Under such circumstances, Expression (19) can be rearranged into a more convenient form for determining:

$$\varphi = \sqrt{1 + \frac{\alpha}{k_m v}} \left(1 - \frac{\rho_s}{\rho_c}\right). \tag{20}$$

Based on experimental findings, it was established that an average value of the resistance coefficient ϕ may be taken as 0.325.

From Expressions (14)-(20), it follows that during the hydromechanical extraction of amber from sandy amber-bearing masses, the ratio of the force resisting the particle's movement through the loose water-saturated medium to the product of the particle's volume, the medium's density, and gravitational acceleration is equal to 0.325.

This provides a relationship for the resistance force acting on an ascending amber particle in a water-saturated medium under vibrational excitation, which depends on the medium's properties and the hydromechanical action parameters [45].

An extremum analysis of Expression (19) indicates that the function reaches its maximum at the point:

$$a_{\max} = \frac{3}{2} \cdot \frac{\phi^2}{1 - \phi^2} \left[1 + \sqrt{1 + \frac{8}{9} \frac{1 - \phi^2}{\phi^2}}\right]. \tag{21}$$

The coordinate of the function's maximum at $\phi = 0.325$ is $a_{\max} = 0.6946$.

However, in addition to having an extremum, Function (19) is defined in the real domain only for values of the Archimedes parameter a that do not exceed the following limit:

$$a \le a_L; \tag{22}$$

$$a_L = \frac{\phi}{1 - \phi}, \tag{23}$$

where:

a_L – the maximum permissible value of the Archimedes parameter.

When a_L, according to Equation (19), the ascent velocity of the amber particle approaches infinity, which contradicts physical reality. At $\phi = 0.325$, the corresponding values are $a_{max} = 0.6946$ and $a_L = 0.48$, i.e., for the current scenario, we have:

$$a_L \le a_{\max}. \tag{24}$$

Accordingly, the maximum determined in Equation (21) lies outside the domain of real values for Function (19).

In studying the vertical motion of an amber particle, the following forces were considered: Archimedes' (buoyant) force, oscillations of the working tool, and the forces of relative resistance from the surrounding medium and viscous drag [46]. In this case, based on the previously obtained results and assuming that the positive direction of the coordinate axis is directed vertically upward, the motion of rising particles can be described by the following Equation:

$$(1 + k_m) m \frac{d^2 z}{dt^2} = m_0 g a + m_0 S f^2 \sin ft - F - F_c; \tag{25}$$

$$m = \rho_s \frac{\pi d^3}{6}; \tag{26}$$

$$m_0 = \frac{\rho_c}{\rho_s} m; \tag{27}$$

$$F = \phi m_0 g a; \tag{28}$$

$$F_c = C_x \rho_c \frac{v^2}{2} \frac{\pi d^2}{4}, \tag{29}$$

where:

z – displacement of the amber particle, m;

t – time, s;

m_0 – mass of the amber-bearing sand within the volume occupied by the amber particle, kg;

S – amplitude of oscillation of the working tool, m;

F_c – viscous drag force acting on the amber particle during its motion through the liquefied medium, N;

d – diameter of the amber particle, m;

C_x – viscous drag coefficient.

Considering the recommendations provided in [47], [48], we assume a Stokes flow regime for amber particles moving through a liquefied fluidized medium. Under these conditions, and according to [49], the dependence of the viscous drag coefficient on the Reynolds number can be expressed as follows:

$$C_x = \frac{26}{\mathrm{Re}_c}; \tag{30}$$

$$\mathrm{Re}_c = \frac{vd}{\nu_c}, \tag{31}$$

where:

Re_c – Reynolds number characterizing the interaction between the amber particle and the liquefied medium;

ν_c – kinematic viscosity coefficient of the liquefied medium.

By substituting Expressions (26)-(31) into Equation (25) and performing the appropriate transformations, we obtain the following equation that describes the motion of an amber particle through a liquefied medium:

$$\frac{d^2\zeta}{d\tau^2} + f_0^2 \frac{d\zeta}{d\tau} = A_0 \frac{\rho - 1}{\rho} + A \sin a\tau. \tag{32}$$

The amber-bearing medium can be considered a fluidized layer when hydromechanical action disrupts the contacts between amber particles and the surrounding matrix and between individual grains of the amber-bearing mass [50], [51]. However, it does not directly influence the amber particles' motion. As a result of this action, particle mobility, and porosity increase, intergranular friction decreases, and the medium acquires a viscosity analogous to the kinematic viscosity of a liquid substance [52]-[54]. In other words, hydromechanical excitation creates favorable conditions for the Archimedes force to act, which enables the buoyant ascent of amber particles to the surface.

Under such conditions, the equation of motion for an amber particle in a fluidized amber-bearing layer under hydromechanical excitation takes the following form:

$$v = \frac{2(1-f)}{39} \cdot \frac{gd^2}{\sigma \nu_g} \cdot \frac{\rho - 1}{\rho}. \tag{33}$$

By solving and transforming Equation (33), the following expression(34) is obtained, which includes the first derivative concerning the variable q:

$$\frac{dv}{dq} = -\frac{\beta + 2\alpha q}{(1-\varepsilon)^2} \cdot \frac{2(1-f)}{39\Delta\sigma} \varepsilon_\omega \frac{gd^2}{\nu_g}. \tag{34}$$

According to Equation (34), the relationship strictly decreases across the entire interval and has no extrema. This implies that, for an adequate description of the investigated processes, it is necessary to consider the variation in density as well as the effective viscosity of the fluidized medium. Such processes are characteristic of treating granular materials in vibrated fluidized beds, boiling beds, and pulsating fluidized layers[55]-[57]. The analysis of established mathematical models that describe hydrodynamic processes in these systems makes it possible to propose two formulas for calculating the dynamic viscosity coefficient of the fluidized medium under consideration [58], [59].

Thus, a combined consideration of Equations (20), (25), (32), (33), and (34) allows us to derive Expression (35), which can be used to determine the ascent velocity of an amber particle in a water-sand mass subjected to hydromechanical excitation:

$$U = \left(1 - \left[\gamma + \beta z + \alpha z^2\right] \varepsilon_\omega\right)^m z^{1.3}. \tag{35}$$

3. Results and discussion

The research has established that the interaction between the turbulent flow and the bed composed of loose amber-bearing rocks is stochastic. This is due to the temporal randomness of turbulent flow velocity and the stochastic distribution of particle diameters within the loose amber-bearing formation.

Given these factors, an algorithm and simulation program were developed for statistical modeling of the turbulent flow's effect on the erodible bed. This algorithm is based on solving the system of differential Equations (1)-(5) using two random number generators that produce stochastic functions $V(t)$ and $d(n)$. These functions have predefined statistical properties corresponding to the natural velocity pulsations and the granulometric composition of the amber-bearing rock mass. The random number generators are implemented based on algorithms that generate random sequences with specified probability density and autocorrelation functions.

Based on this research, a sequential procedure and flowchart were developed to determine the algorithm for identifying detachment conditions of a particle during the hydraulic washing-out of amber-bearing rock, as illustrated in Figure 6. The input data for solving the problem include the statistical characteristics of natural velocity pulsations (probability density function and autocorrelation curve), particle density, and the granulometric curve of the amber-bearing rock. The flowchart in Figure 6 operates as follows: in the first step, the diameter of the test particle $d(n)$ is determined by generating a random number from a sequence whose statistical properties match the granulometric distribution of the studied amber-bearing rock. Next, a second random number, $d(n)$, is generated from the same sequence to determine the protrusion height.

At the next stage, a sequence of random numbers is generated, having the same statistical characteristics as those of natural velocity pulsations. The forces acting on the particle are then determined as functions of the instantaneous natural velocity using the following relationships:

At the next stage, a sequence of random numbers is generated, having the same statistical characteristics as those of natural velocity pulsations. The forces acting on the particle are then determined as functions of the instantaneous natural velocity using the following relationships:

$$P_x(t) = C_x \frac{\pi d^2(n)}{4} \cdot \frac{V_x^2(t)}{2g};$$

$$P_x(t) = C_y \frac{\pi d^2(n)}{4} \cdot \frac{V_y^2(t)}{2g};$$

$$T(t) = \mu d \frac{dV}{dh},$$

where:

C_x and C_y – drag coefficients of the particle in the horizontal and vertical directions, respectively;

$d(n)$ – diameter of the test particle;
$V(t)$ – instantaneous natural flow velocity;
μ – dynamic viscosity coefficient of the fluid.

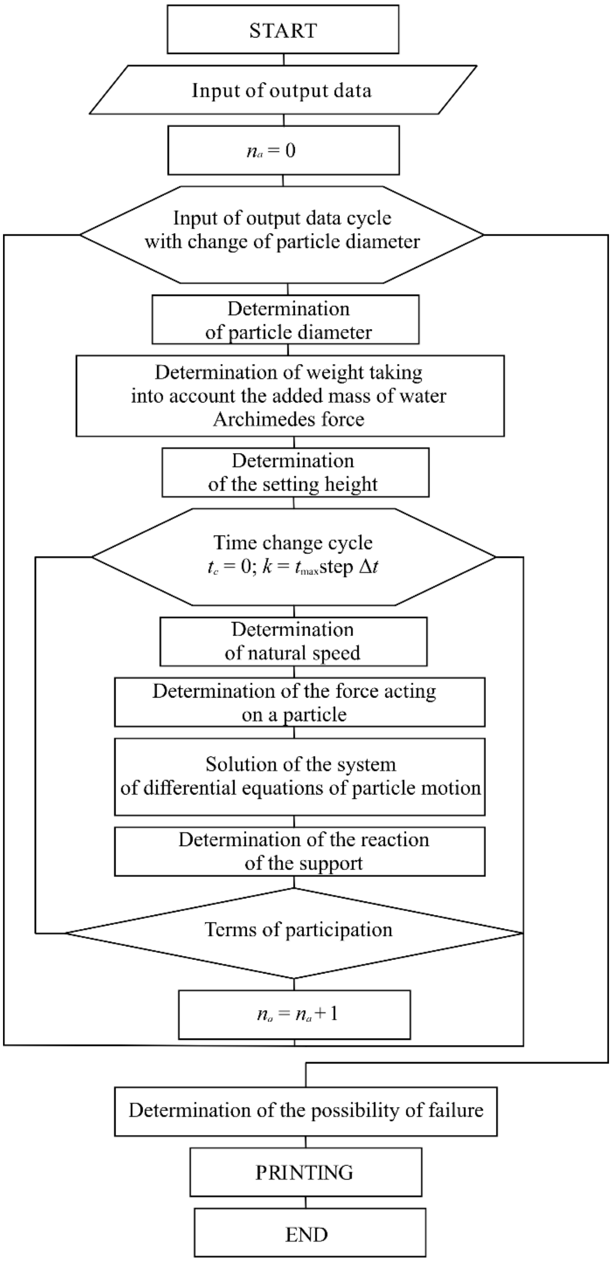

Figure 6. Flowchart of the algorithm for determining particle detachment conditions during hydraulic washing-out of rocks in the hydromechanical extraction of amber

After determining the acting forces, the differential Equations (1)-(5) are solved. This allows the calculation of the particle's rotation angle relative to the support point at a given time and the magnitude of the reaction force from the support.

Next, the detachment conditions are checked. If any detachment criteria are satisfied, the counter of detached particles is incremented by one. After evaluating the first particle, a new particle diameter is generated, and the process repeats.

As a result of the program's operation, the frequency of particle detachment is calculated using the following relationship:

$$P_c = \frac{n_z}{n}, \tag{36}$$

where:
n_z – number of detached particles;
n – total number of tested particles.

The described simulation model of the turbulent flow's influence on particles of a non-cohesive soil forming the erodible bed incorporates several assumptions. These relate to the functional dependencies of the acting forces on natural flow velocities and the idealized shape of the particle. Such assumptions necessitate a series of methodical, numerical experiments to compare the computational results with experimental data.

To this end, numerical simulations were conducted for three particle groups with average diameters d_z = 1.4; 5.0; 10.0 mm. The granulometric composition of the particles was assumed to follow a normal distribution with a standard deviation of 0.3 mm. The time-averaged natural flow velocity was equal to the non-eroding velocity at a reliability level of 0.99, while the velocity dispersion was treated as a variable parameter.

The resulting relationship, described by Equation (35), which expresses the dimensionless ascent velocity of amber particles while accounting for medium saturation with gas bubbles, shows a maximum value close to those obtained experimentally. The velocity also exhibits dependence on the oscillation frequency of the working tool, consistent with laboratory and field experimental findings [60]-[62] (Fig. 7).

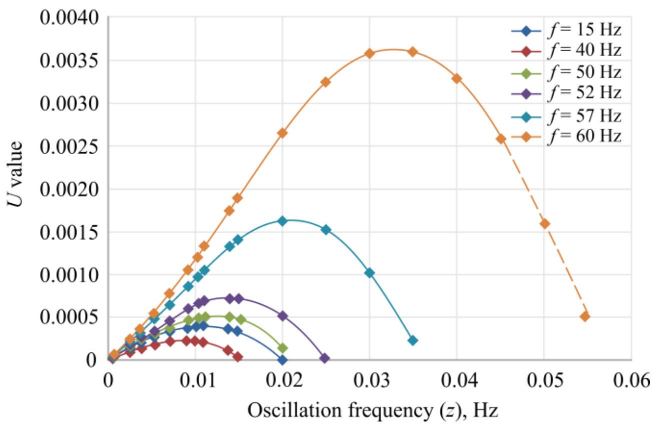

Figure 7. Graphs of the relationship between the variable U and the dimensionless degree of medium saturation with gas bubbles at different oscillation frequencies of the working tool

A comparative analysis of the numerical simulation results, based on Equation (35) for a working tool oscillation frequency of 30 Hz, and the data from laboratory experiments demonstrates a high degree of agreement between theoretical predictions and empirical observations (Figs. 8 and 9). The maximum relative error does not exceed 10%, confirming the proposed model's validity and adequacy for describing the ascent of amber particles in a water-saturated sandy medium under hydromechanical excitation.

Additionally, the extremal properties of Function (35) were studied by analytically determining the conditions for its maximum. Setting the function's first derivative with respect to the corresponding variable to zero confirmed the existence of a maximum point, which is critical for identifying optimal hydrodynamic parameters in the context of amber extraction.

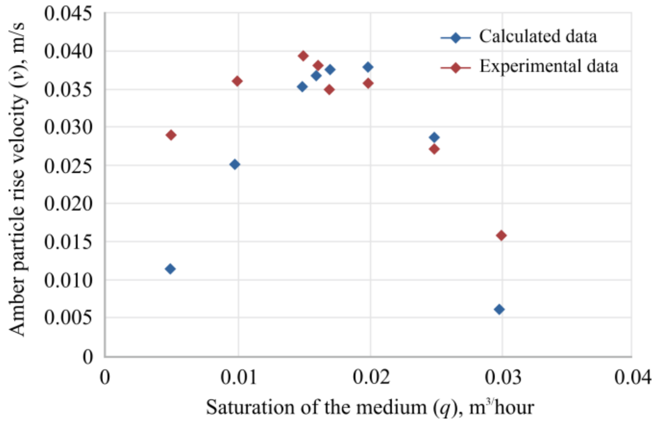

Figure 8. Relationship between the ascent velocity of amber particles and the degree of medium saturation with gas bubbles at a working tool oscillation frequency of 30 Hz

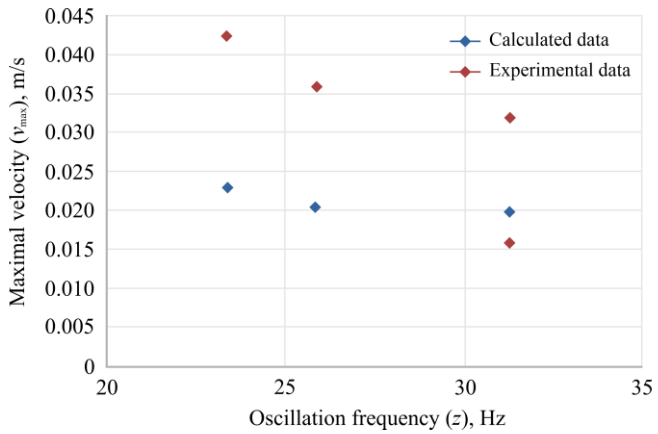

Figure 9. Relationship between the maximum ascent velocity of amber particles obtained via numerical calculation and experimental data as a function of the working tool oscillation frequency

4. Conclusions

Numerical simulations have demonstrated that increasing the degree of heterogeneity in the amber-bearing rock mass leads to a higher probability of particle detachment. For instance, doubling the standard deviation of particle diameters (from 0.2 to 0.4 mm at d_z = 1.4 mm) resulted in a twofold increase in the probability of amber particle detachment, primarily due to the removal of finer particles.

The numerical experiments also revealed an important phenomenon: the detachment behavior of amber particles strongly depends on their diameter. For d_z = 1.4 mm and smaller, particle detachment primarily occurs without rolling or as direct vertical lift-off. In contrast, detachment is almost exclusively by rolling for larger particle diameters. A similar pattern was observed in laboratory experiments. The reliability and stability of the algorithm are further confirmed by its agreement with analytical solutions to the problem.

The amber-bearing medium can be considered a fluidized layer when hydromechanical excitation disrupts interparticle contacts within the matrix and between amber particles and the surrounding mass without directly affecting the motion of the amber particles. This results in increased particle mobility, enhanced porosity of the mass, reduced interparticle friction, and the emergence of viscosity characteristics similar to the kinematic viscosity of a liquid. Hydromechanical excitation creates favorable conditions for the Archimedes force, which drives the upward movement of amber particles toward the surface.

The obtained expression for the dimensionless ascent velocity of amber particles exhibits a maximum value that closely matches the experimentally determined value for the optimal oscillation frequency of the working tool. This aligns with results obtained under both laboratory and field conditions.

A comparison of the simulation results at an oscillation frequency of 30 Hz with laboratory experimental data shows a satisfactory agreement, with a relative error not exceeding 10%, confirming the validity and applicability of the developed model for describing the ascent of amber particles in a water-saturated sandy medium under hydromechanical excitation.

Author contributions

Conceptualization: ZM; Formal analysis: AK; Investigation: YM, AK, VK; Methodology: YM, VM, ZM; Project administration: ZM; Resources: VM, ZM; Validation: ZM; Writing – original draft: YM, VM, AK, ZM, VK; Writing – review & editing: YM, ZM, VK. All authors have read and agreed to the published version of the manuscript.

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Conflicts of interests

The authors declare no conflict of interest.

Data availability statement

The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

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Моделювання процесу гідророзмиву бурштиновмісних гірських порід при видобутку бурштину

Є. Маланчук, В. Мошинський, А. Христюк, З. Маланчук, В. Корнієнко

Мета. Дослідження процесу гідромеханічного видобутку бурштину з піщаних бурштиновмісних порід Волинського Полісся з метою встановлення умов ефективного розмиву та оптимізації технологічних параметрів видобування.

Методика. Основу дослідження становить моделювання гідророзмиву бурштиновмісних порід на прикладі Клесівського родовища. Розроблено математичну модель руху твердих частинок у турбулентному потоці з урахуванням стохастичного характеру швидкості потоку та розподілу частинок за розмірами. Застосовано систему диференціальних рівнянь руху частинок, а також блок-схему статистичного алгоритму визначення ймовірності відриву бурштинових частинок. Лабораторні дослідження проводились на зразках, відібраних із продуктивних піщаних пластів родовища.

Результати. Встановлено, що ймовірність відриву частинок бурштину значно зростає при збільшенні дисперсії швидкості придонного потоку та гранулометричної неоднорідності порід. За результатами чисельних експериментів показано, що дрібні частинки ($d \leq 1.4$ мм) відриваються без перекочування, тоді як більші – здебільшого перекочуються. Отримано залежність ймовірності зриву частинок від параметрів середовища, що дозволяє прогнозувати ефективність гідророзмиву.

Наукова новизна. Запропоновано математичну модель, яка враховує взаємодію випадкових факторів турбулентного потоку з фізичними властивостями частинок. Вперше на основі статистичного підходу визначено критичні умови зриву бурштинових частинок із гірської породи.

Практична значимість. Результати досліджень можуть бути використані для удосконалення існуючих та розробки нових технологій гідромеханічного видобутку бурштину, особливо у складних умовах піщано-глинистих пластів з високим ступенем неоднорідності. Визначення критичних параметрів потоку, що забезпечують максимальну ймовірність відриву бурштинових частинок, дозволяє оптимізувати режими роботи водяних струменів та гідравлічних систем.

Ключові слова: *бурштин, гідророзмив, гірська порода, гідромеханічний спосіб, турбулентний потік*

Publisher's note