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Reliability control of mapping dataset on small subsurface geothermal gradient, a case study in the Zagreb geothermal field, Croatia

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Abstract

Purpose. The research aims to use interpolation methods for mapping, such as Inverse Distance Weighting (IDW), as well as normality tests on geothermal gradient data from the Zagreb geothermal field (ZGF) in the Croatian part of Pannonian Basin System (CPBS).

Methods. The IDW method is applied to original small (8 values) and artificial added (45 values) datasets. The IDW method and a comparison of three geothermal gradient maps (8, 45 and 53 data, respectively) are applied for the ZGF breccia-fractured carbonate reservoir. The maps are compared visually and using mean square error / root mean square error (MSE / RMSE). Dataset is tested on normal distribution (Shapiro-Wilk and Kolmogorov-Smirnov tests, Q-Q plot).

Findings. The insufficient amount of data is the main shortcoming for any subsurface reservoir characterisation. The IDW method has successfully outlined the main reservoir geothermal gradient zones. Increasing dataset with artificial values sampled onto original map (8 values) showed that starting dataset is enough reliable for basic reservoir characterisation. Further reservoir development should be based not on numerous new wells, but on the development of existing wells, including new trajectories and more precise determination of drainage radius, capacity and temperature decline over time.

Originality. For the first time IDW, supplemented with normality tests and artificial sampling based on original small datasets, is applied as a development method in the geothermal reservoir of the CPBS area.

Practical implications. This research is a necessary step in determining the future planning of geothermal reservoir development in the Zagreb urban area. This can be primary or additional approach for a similar reservoir with a small sample, while for a reservoir with a large sample it can show the meaningfulness of choosing an interpolation method.

Keywords: *Croatia Pannonian Basin System, Zagreb, geothermal field, Inverse Distance Weighting (IDW), formal normality tests, Q-Q plot, geothermal gradient, small dataset*

1. Introduction

Spatial analysis is primarily defined as a set of advanced algorithms, mainly interpolation, and is a well-known analytical approach in various geosciences. All spatial interpolation methods allow weighting of data points according to their relative position in comparison to the location being assessed [1]. The aim of this study was to investigate a relatively simple interpolation method by applying it to the real geothermal field of Zagreb and further verify the interpretation with selected statistical analyses, primarily testing for normal distribution of data.

The data used was the geothermal gradient of the wells in the geothermal field of Zagreb. It is one of the seven geothermal fields officially registered in Croatia [2] with several other exploration blocks and regional wells. In almost all situations, the geothermal targets are Mesozoic carbonates and coarse-grained clastics, as well as Neogene (mostly Badenian) weathered limestones and breccias. The majority of larger geothermal sources are considered as targets for electricity generation from medium-temperature geothermal water, applying Organic Rankine Cycle (ORC) [3], [4].

In presented case, the quantity of input data was limited due to the relatively small number of wells. Although small datasets are generally unsuitable for geological mapping [5], [6], this shortcoming was used to assess the accuracy of interpolation by adding "artificial" data and thus to assess the "cost / benefit" of possible new drilling during field development, i.e. whether the existing set of wells is representative of the bulk of the reservoir structure.

A larger number of additional (artificial) points were sampled from the interpolated map of the original (measured) data points, assuming that the original map achieved a representative sample and statistics of the random variable in the research area. Although the variable is random, it was decided to use an approximately regular grid for the new sampling of artificial data so that the new points would not, intentionally or accidentally, be grouped together into clusters, which might influence further interpolation. By mapping and testing the normality of new dataset containing both measured and artificial data, the aim was to confirm the usability of the original map, the appropriateness of the distribution and number of measured points, as well as the expectation of a normal distribution.

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Furthermore, only a larger number of artificially sampled data allowed for meaningful use of normality tests, namely the Shapiro-Wilk and Kolmogorov-Smirnov normality tests. This was important since a large amount of randomly sampled data tends to exhibit the properties of a normal distribution, and such datasets, when interpolating, provide more precise results.

Thus, the main goal of this research was to compensate for the insufficient amount of measured data and give recommendation for determining the reservoir characterization of this lithologically and volumetrically similar geothermal field. The secondary goal was to check the IDW algorithm applicability for presented case, where two datasets exist, one low-sampled and another larger, but mostly artificially created. Keeping of single interpolation algorithms can prevent differences from being interpreted not only statistically, but also as results of different mapping methods applied. Both goals, if successfully achieved, could offer great help in the future planning of geothermal reservoir development in the Zagreb urban area.

2. Methods

2.1. Study area description

Croatia is located at the junction of major European tectonic units: the Alps, the Adriatic Carbonate Platform (ACP, colloquially the Dinarides) and the Pannonian Basin System (PBS) (Fig. 1). Looking at the area of Croatia from the perspective of the use of geothermal resources, it can be divided into two characteristic regions. These are the Croatian part of the PBS (CPBS), which covers the northeastern part of the country, and the Dinarides, which covers the southwestern part [7].

Zagreb Geothermal Field mainly covers southwestern portions of the Sava Depression (Fig. 1). The structural and geomorphological architecture of the Zagreb Geothermal Field is strongly affected by the tectonic position of the Croatian part of the southwestern PBS and Dinarides (Fig. 1). Geological settings of the research area are the result of the polyphase tectonostratigraphic evolution of the Pannonian Basin System (PBS) and the Cretaceous-Paleogene tectonic uplift of the Dinarides [8]-[15].

Tectonically, the area is located at the junction between Eastern Alps, northwestern Dinarides, and Tisza Mega-Unit, covered with Pannonian basin sediments. Zagreb, together with its geothermal field, belongs to the edge area of the CPBS. This area has favorable geothermal properties that are well known, and Figure 2 shows its geothermal gradient.

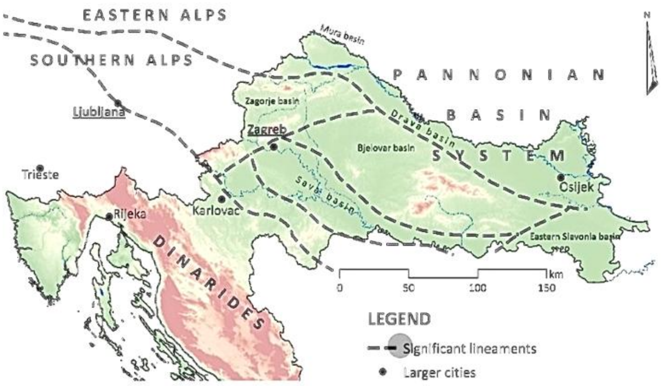

Figure 1. Location of Croatia according to main European tectonic macrounits [7]

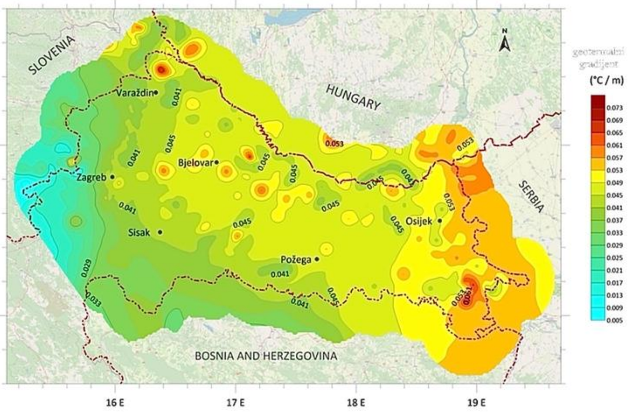

Figure 2. Geothermal gradient in the Pannonian region on Croatia Location of Croatia according to main European tectonic macrounits [16]

The reason for the higher geothermal values is generally thinning of the lithosphere in transtensional areas that belong to the back arch, including deep fault zones. This made it possible to increase heat transfer from deep lithosphere. On contrary, the Adriatic Carbonate Platform unit is characterized by higher heat conductivity in the thicker lithosphere and kilometers of carbonate platform sediments, locally very susceptible to karstification. This also made possible very deep infiltration of meteoric water and the accumulation of very low temperatures in the shallow subsurface [7]. For this reason, geothermal exploration in Croatia is concentrated in the CPBS area.

2.2. Geological settings

There are dozens of natural thermal springs in the Croatian Central Highlands, and most of the geothermal aquifers were discovered in the period from the 1950s to the 1990s due to extensive research and drilling for hydrocarbons. One example is the Zagreb geothermal aquifer. Although many wells that could have been promising for geothermal water extraction were technically abandoned because they gave negative results on hydrocarbon content, detailed logs of these investigations have nevertheless remained, which constitute a large set of existing information relevant to geothermal prospecting.

The Zagreb geothermal field (ZGF) was discovered during hydrocarbon prospecting in the 1960s. It has an area of about 54 km^2 and is located in the city of Zagreb [17]. Given the large number of potential users, the ZGF is an example of an available resource that can be well utilized. At the same time, due to its relatively high number of measurements and years of production, it is a geothermal reservoir that is suitable for testing methodologies and work processes for the development of similar reservoirs.

The ZGF was developed as a geothermal field, and exploration and drilling continued until 1988 [17]. Although wells in the outer part of the field are also used, such as the Lučanka and Nedelja wells (Fig. 3), the main development is represented by two so-called technological systems with the highest measured temperatures and geothermal gradients, Mladost and KBNZ, in the central part of the field [18].

Figure 3 shows the geothermal gradient of the reservoir top, the self-outflow capacity and the well mouth temperature for each well for which this data is available. Figure 4 shows a schematic cross-section of the ZGF area, whose profile is marked in Figure 3.

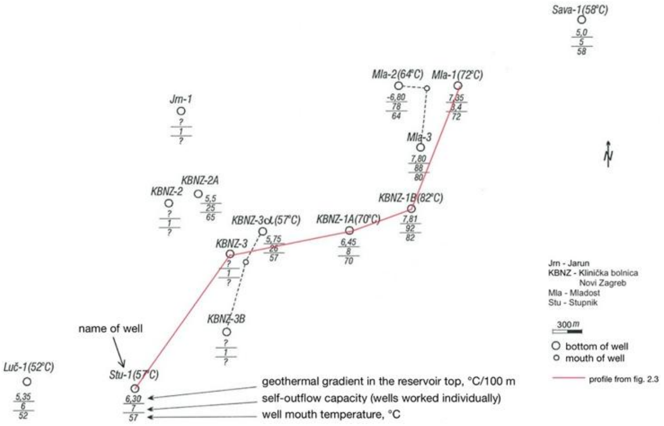

Figure 3. Locations of wells in the Zagreb geothermal field [19]

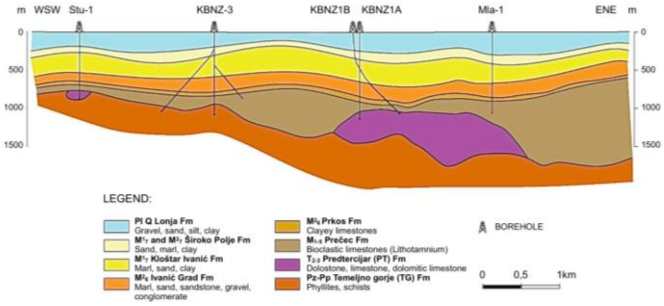

Figure 4. Schematic geological section along (Figs. 2, 3) the Zagreb geothermal field [7]

The research area is characterized by a thick sedimentary succession of the PBS (based on drill-hole data, from 800 m up to more than 3.5 km) and different rocks in the basement. Basement is usually represented by Middle to Upper Triassic dolomites, Paleogene and Cretaceous sediments and crystalline basement. The Middle and Upper Triassic dolomites ($T_{2,3}$) that represent one of the geothermal aquifers in the research area are drilled by a few drill holes at different depths (from 790 to 4074 m). These rocks are outcropping in the Žumberak and Medvednica Mountains (Fig. 5), so their structural and hydrological properties are available for studies and outcrop analogue analysis. This carbonate succession is formed on the passive margin of the Neotethys Ocean on the large carbonate platform [13], [20]. The large depositional paleoenvironment resulted in a very large lateral and vertical distribution of $T_{2,3}$ dolomites with similar sedimentary properties. The thickness of the Triassic carbonate succession often exceeds thousands of meters [20], [21], whereas in the Žumberak area, the total thickness exceeds 2250 m [22]. Besides the mentioned structural architecture of the subsurface, which is governed by regional structural relations, the architecture of the Zagreb-Karlovac geothermal area was further influenced by the evolution of the Miocene-Quaternary Pannonian Basin System associated with deformation processes that resulted in the formation of several tenths of kilometers long NW and NE – striking faults / faults (i.e., North Medvednica Fault – NMF; Sava Fault – SF, Kašina Fault – KF, and Karlovac depression boundary fault (Fig. 2) in the area.

Marine flooding of the CPBS during the Badenian period [23] resulted in a transition from fluvial and lacustrine environments to predominantly marine environments [8], [10], [24] and the deposition of shallow-water limestones characterized by algae Lithothamnium, which represents second geothermal aquifer.

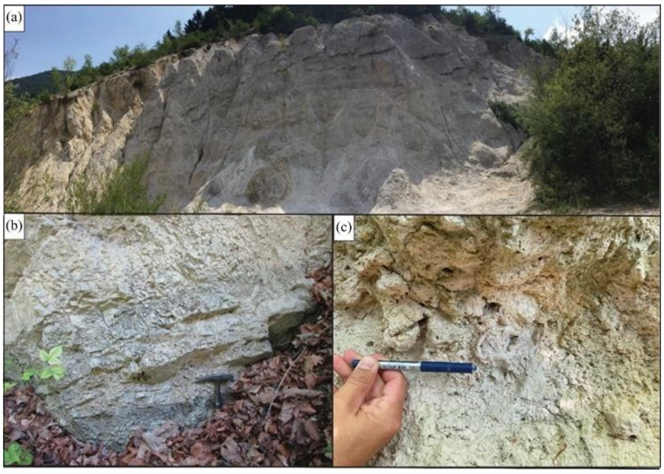

Figure 5. Photographs of the geothermal aquifer outcrops in the area of Žumberak and Medvednica Mts: (a) T_3 stromatolite dolomites in Plešivica (Žumberak Mts); (b) T_3 fractured dolomites in Slapnica valley (Žumberak Mt.); (c) M_4 Badeninan Lithothamnium limestones (Medvednica Mt.)

Very porous, shallow-water Lithothamnium limestones represent the main part of the geothermal aquifer, but also transgressive members like conglomerates and sandstones can also have aquifer characteristics. The main characteristic of the Badenian sequence is frequent lateral and vertical transitions into different lithofacies. The geometry and structural position of this aquifer primarily depend on the depositional environment and sedimentation mechanism. Recorded water temperatures reach 80°C in the Mladost technological system [18] and in the KBNZ system [17]. In the most wells, geothermal fluid is produced from bioclastic limestones. However, both lithologies are considered as single hydrodynamic unit. Figure 6 shows well temperatures at various depths.

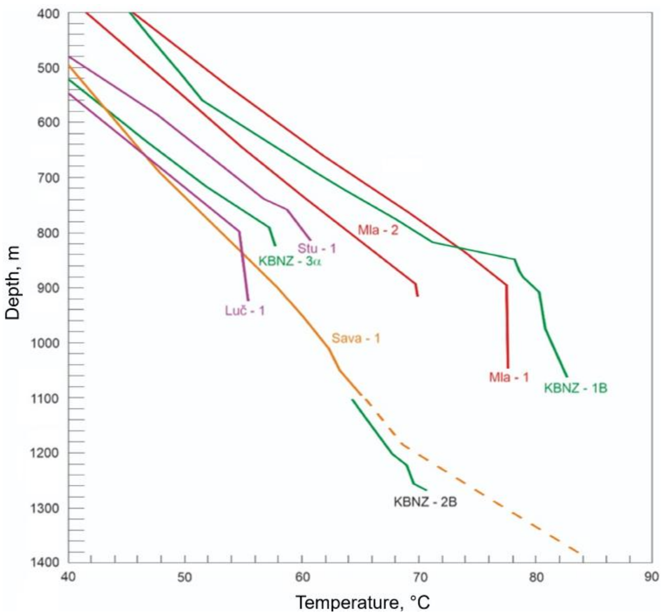

Figure 6. Temperature lines at different well depths [19]

2.3. Inverse distance weighting

By selecting an appropriate interpolation method using the input data, the values of the selected variables were estimated at the points of the interpolation network, where measurements were not made. In this case, the method of

IDW was chosen. This method is suitable if the input data are points that are not strongly grouped [25], which is the case here. The IDW method is also suitable for cases where we have a smaller quantity of input data, which is an additional reason for choosing this method. Another reason is that the IDW method does not require the input dataset to have a normal distribution, like geostatistical interpolation methods [6].

At the location where the value of a variable is estimated, the influence of each point is inversely proportional to its distance from this location. The value estimate includes all points within a certain radius from the location of the estimate [25]. The following Expression (1) describes the calculation by which we obtain the estimate of the values of the variables at the selected points:

$$Z_{UI} = \frac{\frac{Z_1}{d_1^p} + \frac{Z_2}{d_2^p} + \ldots + \frac{Z_n}{d_n^p}}{\frac{1}{d_1^p} + \frac{1}{d_2^p} + \ldots + \frac{1}{d_n^p}}, \qquad (1)$$

where:

n – number of input data points;

Z_{UI} – estimated value;

d_i – distance to the "i-th" location;

p – power of distance;

Z_i – measured values at "i-th" location (I = 1,…, n).

The distance exponent significantly affects the result achieved by the IDW method, and the most commonly used value is 2, which relatively successfully balances the results between the excessive influence of distant measurements and the excessive emphasis on measured values by a series of concentric isolines [25].

2.4. Root mean square error

In order to assess the accuracy and precision of the results obtained by the selected interpolation method, it is necessary to perform a numerical calculation of the error. There are several such methods, and here two are described, the calculation of which was available in the interpolation program used (Golden software Surfer).

The first of these methods is root mean square error (RMSE). This method measures the average difference between the predicted values of a statistical model and the actual values [26]. Mathematically, this is the standard deviation of the residual values, or the distance between the regression line and the data points. RMSE quantifies how closely the measured data are clustered around the predicted values $\bar{y}$ obtained by interpolation. As the data points get closer to the regression line, the model has a smaller error, lowering the RMSE. An interpolation model with a smaller error produces more precise and accurate predictions. The value of RMSE ranges from zero to positive infinity, and the unit used is the same as the dependent variable. An RMSE value of 0 indicates that the predicted values y match the actual values $\bar{y}$ perfectly, but this is generally not the case in practice. Low RMSE values suggest that the model fits the data well and has predictions that are more accurate. Conversely, higher values indicate a larger error and less accurate predictions. The RSME Formula (2) looks familiar because it is the same as the standard deviation formula. It measures the dispersion of measured values around predicted values.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}\left(y_1 - \bar{y}_i\right)^2}{N}}, \qquad (2)$$

where:

N – number of input data points;

$y_1 \ldots y_N$ – actual values (1, …, N);

$\bar{y}_i \ldots \bar{y}_N$ – predicted values (1, …, N) obtained by interpolation.

In the RMSE equation, the difference between the measured and estimated value at the same location represents the residual, or "error" between the actual and predicted value. The squares of the residuals are summed, and then the sum is divided by the total number of input data points to obtain the average of squared residuals. Finally, RMSE is calculated by taking the square root of this value.

Although RMSE is a relatively simple metric that provides a clear interpretation of the overall error of the model, it does have its weaknesses. For instance, one of its weaknesses is its oversensitivity to larger errors, which is to say that RMSE gives disproportionately higher weights to outliers.

2.5. Mean square error

The second method for calculating error is the mean squared error (MSE). Similar to RMSE, the MSE method measures the average difference between the predicted values of a statistical model and the actual values [27]. When the model has no error, i.e. when all predicted values $\bar{y}$ are equal to the actual values, the MSE is equal to zero. As the error of the model increases, the MSE value increases. The Formula (3) for MSE is as follows:

$$MSE = \frac{\sum_{i=1}^{N}\left(y_1 - \bar{y}_i\right)^2}{N}, \qquad (3)$$

where:

N – number of input data points;

$y_1 \ldots y_N$ – actual values (1, …, N);

$\bar{y}_i \ldots \bar{y}_N$ – predicted values (1, …, N) obtained by interpolation.

The calculation of MSE is similar to variance, i.e. the square of the standard deviation. Squaring the differences in the MSE formula has multiple reasons. It eliminates negative difference values and thus ensures that the MSE value is always greater than or equal to zero. Only a perfect error-free model produces an MSE equal to zero, and this does not happen in practice.

As with RMSE, one of the weaknesses of the MSE method is that squaring increases the impact of larger errors in the event that there are outliers. Taking the square root of the MSE value, the RMSE value is obtained, which uses the natural data units. In other words, MSE is analogous to the variance, while RMSE corresponds to the standard deviation.

2.6. Shapiro-Wilk test

Once the data is interpolated and the precision of the interpolation is assessed by numerical calculation of the error, the results need to be further analyzed. One way to achieve this is to take the set consisting of both the measured and the artificially sampled data and check whether such a large set is normally distributed. Two tests are used for this purpose, the first of which is the Shapiro-Wilk test.

The Shapiro-Wilk test checks whether the normal distribution model fits the input values. This is usually the most powerful test for normality. The test calculates two values: W and p. When the test is performed, the W statistic is always positive and represents the difference between the normal distribution model and the observed data. The larger the W statistic, the more likely it is that the model is incorrect [28]. The W statistic is derived from the correlation between the input data and the corresponding normal results. A W value close to 1 indicates that the data is normally distributed [28].

The null hypothesis for this test is that the data is normally distributed. The test calculates p-value and compares it to selected α value. The most used value is $\alpha = 0.05$, i.e. the 95% degree of significance. If p-value is less than α value, then the null hypothesis that the data is normally distributed is rejected. Otherwise, if p-value is greater than α value, the null hypothesis is considered confirmed.

A web calculator [29] is used to implement the Shapiro-Wilk test. The calculator allows the entry of α value, and the default value $\alpha = 0.05$ is used. An option to exclude outliers is also offered, which is not necessary in this case, because extreme values are not present in the input dataset. The paper also supplements the formal tests with Q-Q plot calculation.

2.7. Kolmogorov-Smirnov test

The second test for normality used is the Kolmogorov-Smirnov test. This is not the most rigorous test, but its advantage is that it can be used for any distribution, i.e. the test is nonparametric [28]. However, it is commonly used as a test of normality by comparing a known hypothetical, in this case normal, distribution with the distribution generated by the input data, i.e. the empirical distribution function.

The Lilliefors test is a corrected version of the Kolmogorov-Smirnov test for normality that generally approximates the test statistics distribution more accurately. Many statistical packages combine these two tests as the "Lilliefors corrected" Kolmogorov-Smirnov test.

The Kolmogorov-Smirnov test tests two hypotheses. The null hypothesis is that the data is normally distributed, and the alternate hypothesis is that at least one value does not fit the normal distribution. The Kolmogorov-Smirnov test calculates a statistic D that measures the greatest distance between the empirical function $F_{data}(X)$ and the theoretical function $F_0(X)$, measured vertically. The Formula (4) is as follows:

$$D = \sup_x \left| F_0(x) - F_{data}(x) \right|, \tag{4}$$

where:

$F_0(X)$ – cumulative distribution function of the hypothesized (i.e. normal) distribution;

$F_{data}(X)$ – empirical distribution function of the input data.

If the value of D is greater than the critical value, then the null hypothesis is rejected. The critical value for the D statistic is a fixed value that depends on the number of input data points and the chosen value of α. As mentioned in section 2.5, the most commonly used value is $\alpha = 0.05$, which indicates a 95% degree of significance. As with the Shapiro-Wilk test, if the p-value is less than α value, then the null hypothesis that the data is normally distributed is rejected. The smaller the p-value, the more support there is for the alternate hypothesis that at least one value does not fit the normal distribution.

Similarly, as for the Shapiro-Wilk test, a web calculator [30] is used for the Kolmogorov-Smirnov test. A standard value of $\alpha = 0.05$ is used, and outliers are included because the input dataset does not contain them. Among the relevant statistical parameters calculated by the calculator are the D statistic and p-value, as well as the graphical representations of normality including Q-Q plot.

3. Mapping and statistical results of input data analyses

In this section, the methods described in section 2 are implemented with data sampled in the ZGF area. It is explained how these data points were sampled, after which their mapping and statistical data analysis are presented.

3.1. Input data and mapping

The data used was the temperature gradient for each of the 13 wells in ZGF. Since certain wells are located at the same location or are very close to each other, in such cases they were considered to be the same point. Therefore, a total of 8 data points were obtained from the 13 wells. Table 1 shows the coordinates and the temperature gradient values for the 8 measured data points. Figure 7 shows the result of interpolation for these data points.

Table 1. Input (measured) data

Well name (real point data)	x-coordinate	y-coordinate	Temperature gradient (°C / 100 m)
A	455766.2	5070239	6.49
B	453988.4	5070741	4.37
C	454581.4	5069992	4.26
D	456893	5071658	6.41
E	450763.8	5068553	4.92
F	458225.1	5072293	5.1
G	453460.4	5068882	5.86
H	454187.9	5072059	5.36

On the map in Figure 7, 45 new (artificial) points were sampled, relatively regularly distributed. Given the relatively small quantity of input data, a larger number of additional points were sampled to assess the precision of the interpolation performed and to enable additional analysis of a statistically representative amount of data

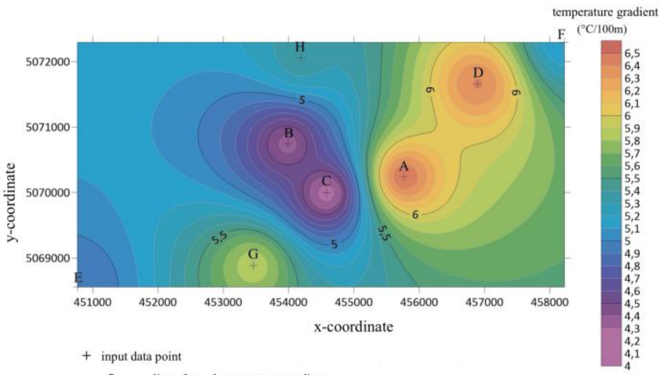

Figure 7. Temperature gradient map in the geothermal reservoir of ZGF obtained using IDW with Table 1 data

If additional data is interpolated using the same method and then a numerical calculation of the error shows that the increase in error is not significant with the addition of new points, this would indicate the relative accuracy and precision of the original interpolation. In this case, additional data is useful because it can be used for further statistical analysis such as tests for normality, which require the input of a larger number of values. Furthermore, these points are sampled in a relatively regular pattern, since there is no reason to conclude that points should be more densely clustered at any particular location, and therefore a regular pattern is the most sensible.

Readings of the temperature gradients at the artificially sampled points were taken from the map and were henceforth considered as measured data. Table 2 shows the coordinates and the temperature gradient values for the 45 artificially sampled points. With this new data, the IDW method was repeated. Figure 8 shows the result of this interpolation with the input of 45 artificially sampled points, and Figure 9 shows the same interpolation but with the input of all 53 points.

Table 2. Artificial data presented as larger dataset, where each point has coordinates and value recorded from map in Figure 7

Artificial data no.	*x*-coordinate	*y*-coordinate	Temperature gradient (°C / 100 m)	Artificial data no.	*x*-coordinate	*y*-coordinate	Temperature gradient (°C / 100 m)
1	451334.9	5071652	4.98	24	454931.8	5070371	5
2	452247.3	5071803	5.03	25	455942.3	5070280	6.5
3	453235.1	5071766	5.08	26	456726.5	5070265	6.44
4	454162.6	5071788	5.23	27	457254.4	5070250	6.31
5	455029.8	5071713	5.56	28	457767.1	5070152	6.16
6	455836.7	5071735	6.05	29	457850.1	5069782	6.15
7	456568.1	5071735	6.38	30	457111.1	5069797	6.31
8	457080.9	5071811	6.2	31	456364.5	5069722	6.35
9	457661.6	5071735	5.82	32	455633.1	5069722	5.98
10	451319.8	5070846	4.98	33	455067.5	5069714	5.22
11	452096.5	5070913	5.01	34	454539.7	5069714	4.64
12	452986.3	5070898	4.97	35	453800.7	5069677	4.97
13	453906.3	5071034	4.8	36	453205	5069654	5.23
14	455127.9	5070830	5.44	37	452812.9	5069647	5.25
15	456032.8	5070815	6.36	38	452202	5069616	5.19
16	456862.2	5070838	6.43	39	451719.4	5069564	5.11
17	457631.4	5070913	6.12	40	451417.8	5068915	5.1
18	451206.7	5070265	4.99	41	452322.7	5068915	5.39
19	451923	5070182	5.06	42	453016.5	5068975	5.62
20	452571.5	5070318	5.06	43	453491.5	5068968	5.76
21	453385.9	5070333	4.9	44	454072.2	5068960	5.47
22	454049.5	5070288	4.52	45	455210.8	5068968	5.63
23	454441.7	5070325	4.44				

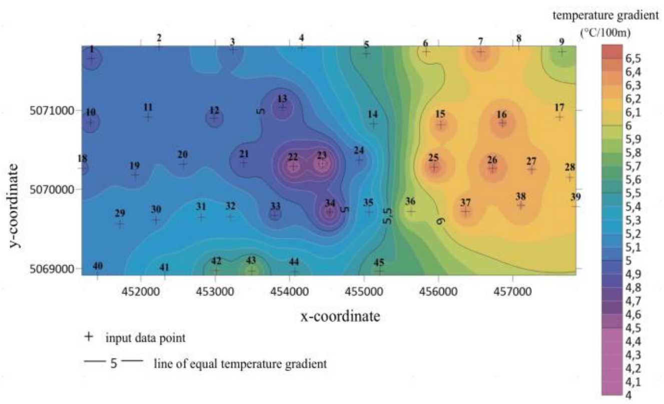

Figure 8. Temperature gradient map in the geothermal reservoir of ZGF obtained using IDW with Table 2 data

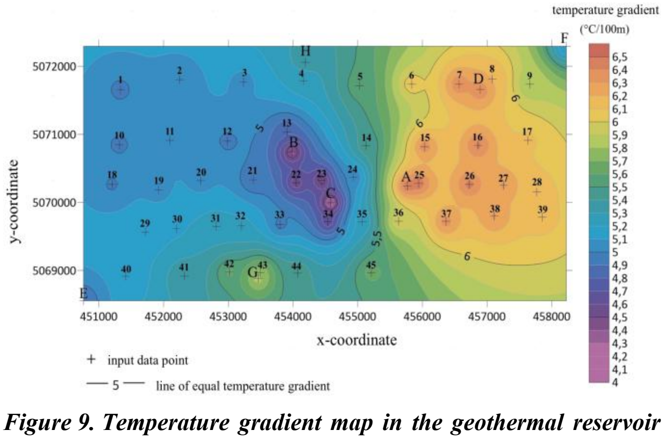

Figure 9. Temperature gradient map in the geothermal reservoir of ZGF obtained using IDW with Tables 1 + 2 data

Comparing Figures 8 and 9, it is evident that there is no great difference in the two interpolations, and the largest deviations are found around the locations of the original 8 points. Such a result is reasonable to expect because the artificially sampled points are plotted in a relatively regular pattern, and the addition of 8 original points disrupts this regular pattern. Nevertheless, the similarity of these two interpolations indicates the relative precision of the initial interpolation (Fig. 7), which is confirmed by the calculation of the numerical error. Using the Surfer program, the RMSE and MSE were calculated for all three interpolations. Table 3 shows the values of these errors.

Table 3. Numerical errors of interpolated maps

Error type	Figure 7	Figure 8	Figure 9
RMSE	5.405	5.524	5.506
MSE	29.217	30.513	30.318

It is evident that adding 45 new points increases the error, as expected, but this increase is not significant. The interpolation in Figure 8 has an increase in RMSE of 2.20% and MSE of 4.44% compared to Figure 7, even though it contains 5.625 times more data points. Similarly, the interpolation in Figure 9 has a 1.87% increase in RMSE and a 3.77% increase in MSE compared to Figure 7, despite containing 6.625 times more data points. Therefore, we can conclude that both interpolations are relatively accurate. The interpolations in Figures 8 and 9 have almost identical error values, although the one in Figure 9 is slightly lower. This can be explained by observing that the interpolation in Figure 9 also contains data from Figure 7, while the interpolation in Figure 8 contains a completely different dataset compared to Figure 7.

3.2. Statistical analysis

After interpolation, the data was statistically analyzed using two different tests for normality. For the set of all 53 data points from Tables 1 and 2, the Shapiro-Wilk and Kolmogorov-Smirnov tests were utilized to check whether it is normally distributed. The Shapiro-Wilk test gives the result $W = 0.9315$ and p-value of 0.004653. The Kolmogorov-Smirnov test gives the result $D = 0.1456$ and p-value of 0.006796. Since the p-value is less than 0.05 in both tests, we conclude that this dataset is not normally distributed. Figure 10 shows the Q-Q plot for this dataset.

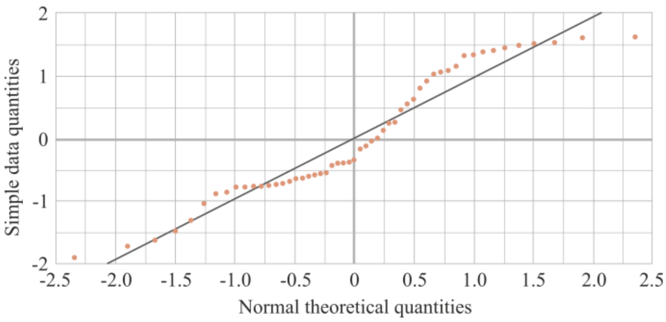

Figure 10. Q-Q plot for data from Tables 1 and 2

4. Discussion

In this paper, the relative accuracy and precision of the initial interpolation (Fig. 6) are successfully demonstrated. This can be concluded from the comparison with the maps in Figures 7 and 8 and from the numerical calculation of the error, since the addition of a much larger quantity of input data causes only a small increase in the error. Although the analysis was successful in this aspect, it also has certain shortcomings.

The results of the Shapiro-Wilk and Kolmogorov-Smirnov tests indicate that the dataset is not normally distributed, although the construction of the Q-Q plot showed that such an interpretation is marginal. However, rejection by a formal test is a potential problem because many statistical and interpolation methods use the assumption of normality. Furthermore, larger sets of randomly sampled geological variables often show normality. However, physically the geothermal gradient of an area should not exhibit such a property, because the gradient through the rocks does not flow homogeneously in any direction, but "pointwise", or most often "concentrically".

Furthermore, the increase in numerical error of maps with more data is relatively small after adding artificially sampled points, as can be seen in Table 3. In general, linear interpolation models show greater accuracy and precision the smaller the error, but since only one interpolation method (IDW) was used, it is not possible to compare such errors on two maps with the same input dataset, interpolated by other methods. Also, the deviation of the input data from the normal distribution limits the number of interpolation methods that can be reliably used in this case, i.e. all geostatistical ones should be excluded from future considerations. However, as a comparative method, it would be better to try algorithms such as Radial Basis Function, which have already been compared for small, deep, geological datasets in the CPBS [27].

5. Conclusions

All previously discussed brings the consideration back to the initial limitation of the set of only 8 measurements, which in any case is an insufficiently representative set, and therefore difficult to analyze using any statistical and interpolation methods. Adding artificially sampled points only partially reduces this problem, because this step also contains bias, but does not eliminate the inherent shortcomings that make it impossible to describe the variability of the geothermal reservoir. Nevertheless, it is possible to draw several solid conclusions and recommendations for further research into the ZGF reservoir, as well as lithological and volumetrically similar geothermal reservoirs elsewhere. These are:

– the geothermal field of Zagreb has an area of 54 km^2 but is only tapped from a small number of wells in that area, located approximately in the central part of the reservoir (Mladost and KBNZ wells);

– in total, the geothermal gradient was measured in only 8 wells, which is a very small set for any statistical analysis and marginally small for interpolation;

– the map obtained from 8 wells proved to be sufficiently reliable for estimating the temperature and yield of the reservoir, which was proven by comparing the solution with possible maps when the number of data points on them was 45 and 53. The visual difference of the solution (especially the maxima and minima) was not significant;

– by artificially increasing the dataset by relatively regular resampling on the gradient base map (from 8 data points), sets were obtained whose distributions could be tested formally (Shapiro-Wilk and Kolmogorov-Smirnov) and informally (Q-Q diagram). In all cases, it was not confirmed that the dataset was normally distributed, although the rejection was borderline (Q-Q plot);

– the lack of a normal distribution justified the use of the IDW interpolation method, as it is mathematically simpler, and normality is not a strict formal condition (as, for example, in Kriging);

– mapping, interpretation and statistic analysis were also influenced by reservoir lithology that is mostly represented with Badenian breccia, and in deeper parts with fractured Triassic carbonates;

– although production is performed exclusively from breccia, reservoir fluid hydrodynamical behavior is determined by both lithological units, where all petrophysical properties are stochastically distributed and very hardly predictable;

– consequently, further development of the reservoir should not include the construction of a large number of new wells or channels (planned mainly in a regular network across the reservoir), because the basic thermal properties of the same ones are already well established from the 8 existing ones;

– as shown in figures, and by calculating the corresponding errors and analyzing the data for normality, a significant increase in data will not lead to a significant change in the shape of the isolines on these maps, nor a large increase of numerical errors.

Author contributions

Conceptualization: MJ, IP, TM; Data curation: MJ; Formal analysis: MJ; Funding acquisition: TM; Investigation: MJ, IP, TM; Methodology: MJ, TM; Software: MJ; Supervision: TM; Validation: MJ, IP, TM; Visualization: MJ; Writing – original draft: MJ, IP, TM; Writing – review & editing: MJ, IP, TM. All authors have read and agreed to the published version of the manuscript.

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Conflicts of interests

The authors declare no conflict of interest.

Data availability statement

The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

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Контроль надійності картографування набору даних про малі підземні геотермальні градієнти, тематичне дослідження на Загребському геотермальному родовищі, Хорватія

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Мета. Використання методів інтерполяції, а саме зворотно-зважених відстаней (ЗЗВ) та тестів на нормальність даних геотермального градієнта для надійності картографування набору даних в умовах Загребського геотермального родовища (ЗГР) в Хорватській частині Паннонської басейнової системи.

Методика. Метод ЗЗВ застосовується до вихідних малих (8 значень) та штучно доданих (45 значень) наборів даних. Метод ЗЗВ та порівняння трьох карт геотермальних градієнтів (відповідно 8, 45 і 53 даних) застосовано для брекчійно-тріщинуватого карбонатного покладу ЗГР. Карти порівнюються візуально та за допомогою середньоквадратичної похибки / кореневої середньоквадратичної похибки (MSE/RMSE). Набір даних перевіряється на нормальний розподіл (тести Шапіро-Вілка та Колмогорова-Смірнова, Q-Q графік).

Результати. За результатами досліджень метод ЗЗВ успішно окреслив основні зони геотермального градієнта покладу. Збільшення набору даних за допомогою штучних значень, які були нанесені на оригінальну карту (8 значень), показало, що початковий набір даних є достатньо надійним для базової характеристики покладу. Визначено, що подальша розробка покладів повинна базуватися не на численних нових свердловинах, а на розробці існуючих свердловин, включаючи нові траєкторії та більш точне визначення радіусу дренування, продуктивності та зниження температури в часі.

Наукова новизна. Вперше метод ЗЗВ, доповнений тестами на нормальність і штучною вибіркою на основі оригінальних невеликих наборів даних, застосовується як метод розробки геотермального родовища на ділянці Паннонської басейнової системи Хорватії.

Практична значимість. Проведене дослідження є необхідним кроком у визначенні майбутнього планування розробки геотермального покладу у міській зоні Загреба. Це може бути основним або додатковим підходом для подібного покладу з невеликою вибіркою, в той час як для покладу з великою вибіркою це може показати значущість вибору методу інтерполяції.

Ключові слова: *Паннонська басейнова система Хорватії, Загреб, геотермальне родовище, метод зворотно-зважених відстаней (ЗЗВ), формальні тести на нормальність, Q-Q графік, геотермальний градієнт, невеликий набір даних*

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