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Integration of stability factors A, B and C on the Mathews Stability Graph into a mining-level optimization algorithm

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Abstract

Purpose. This study aims to enhance the optimization approach by integrating stability analysis using Mathews Stability Graph into stope mining-level optimization algorithm.

Methods. The programming language is employed to integrate the Mathews Stability Graph into the stope mining-level optimization algorithm at the preliminary optimization stage, providing dimensional constraints based on rock conditions. Algorithm validation is conducted using three scenarios reflecting rock conditions in the block model: fixed stope dimensions with a maxi-mum stope size, fixed stope dimensions with a minimum stope size, and variable stope dimensions based on the proposed algorithm. Additionally, to validate the stability of the stope in the optimization algorithm, the stability of each stope wall is confirmed by back plotting on a stability graph.

Findings. The algorithm manages to create a stope design that complies both with geotechnical and economic aspects, based on the data provided in the synthetic block model.

Originality. Optimal stope design is often determined by the stope's economic parameter, whereas geotechnical variables, easily available in the block model, are neglected. The proposed algorithm aims to include stability analysis using the Mathews Stability Graph into the stope mining-level optimization algorithm.

Practical implications. The method was successfully tested using data from a block model simulating the conditions of a real ore body in Indonesia. In addition, the method may be used by mine planners during the early stage of feasibility assessment.

Keywords: *stope optimization, Mathews Stability Graph, stope stability, stope layout, algorithm*

1. Introduction

The underground stope method is widely used in mines worldwide due to its flexibility in operation, ability for selective mining, and relatively lower geotechnical risks compared to the caving method. However, determining an optimal mining plan, commonly referred to as stope layout, remains a persistent challenge due to the complexity of variables and parameters. One of the important variables in mining optimization, especially in the technical aspect, is the mining level [1]-[3]. The ideal mining level is determined based on the capabilities of stopes installed at each level to maximize profits while ensuring the stability of the mining opening throughout the entire mining operation [4]. When determining the mining level, it is important to understand how this factor will affect the stope shape.

Mining levels and stope sizes are interrelated characteristics. Larger stope sizes lead to fewer mining levels, which in turn increases productivity by reducing the number of mining cycles. Smaller stope sizes result in more levels, enabling selective mining and flexibility. The most economically favorable combinations of mining levels are often selected. However, stope sizes and mining levels are largely dependent on rock conditions. Rock condition assessment is crucial in stope design as it determines the viability of the stope and contributes to mine safety.

Various methods have been used to analyze the stability requirements of stopes [5]-[8]. Two major empirical studies regarding stope stability analysis are the Rock Mass Rating (RMR) and the Q-System. The Mathews Stability Chart is a widely recognized empirical method based on the Q-System that is accepted as an industry standard for suggesting stope size in the early phase [9]. The Mathews method can be used to determine the size of the stope wall by examining the rock mass class (Q′) along with three other factors: the induced stress conditions on the stope wall (factor A), the orientation of the geological structure with respect to the stope wall (factor B), and the orientation of the stope wall with respect to gravity (factor C). Studies on the stability of the Mathews Chart have also developed rapidly, especially in underground mining with the open stope method [10]-[13]. In practice, the mine planner uses empirical stability analysis in stope design independently from the optimization process. The acceptable stope geometry recommendation is given to the mine planner as the basis for conducting stope optimization and design. While this method was viable, it became tedious as the geomechanical data grew with the progression of mining. Rede-

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sign is a common practice as new recommendations are delivered based on the newly provided geomechanical data. To address these issues, stope optimization has been significantly developed in recent years.

Based on current developments, stope optimization studies can be divided into two major perspectives: algorithmic development in stope optimization to significantly reduce analysis time, and stope optimization from a geomechanical perspective that focuses on improving stope stability analysis. These two studies have their own novelties; however, integration between them is scarce, despite the massive need for such integration to achieve a truly optimal design [14].

The early development of stope optimization algorithms in underground mining was dominated by exact algorithms. An exact algorithm is a mathematical approach that generates a precise solution, ensuring that the optimal solution is achieved. As the mathematical formulation becomes increasingly complex, the time needed to find the final solution grows significantly, leading to reliability issues. However, the simplicity of this approach was a major advantage, as it was widely used in the early development of stope optimization. The stope optimization algorithm was first introduced by Riddle [15], who proposed dynamic programming for a block caving case study to determine the optimum area within the block model. The approach has been proven to be easily implemented, although it is limited to two-dimensional case studies and requires manual design to meet practical demands. In 1984, Deraisme et al. [16] introduced a geostatistical approach to define the optimum area within a block model in 2D sections. While successfully creating a 3D representation of the optimum area by accumulating 2D sections, the approach was limited to the cut-and-fill method and did not consider mine economics; thus, it failed to determine the global optimum. Ovanic and Young [17] introduced branch and bound algorithms to address the limitations of integer programming performance in large case studies. However, limitations in one-dimensional cases and reliability issues in large cases remain challenges for this approach.

To address reliability problems, heuristic algorithms [18]-[22] have been developed. Heuristic algorithms are capable of generating a solution in a reasonable amount of time, although the results are not always optimal, thus addressing reliability issues in large-scale cases such as stope optimization. Octree Division [23], one of the earliest algorithms developed in this category, can transform a 3D geological model into mining reserves by enforcing mining and economic constraints in its optimization. Floating Stope algorithms [24] were among the few algorithms subsequently adopted in commercial software. Their simplicity and well-known structure, similar to optimization algorithms used in surface mining, proved helpful in determining the optimum area during the early phase of feasibility assessment. However, issues with overlapping stopes led to the development of algorithms to overcome this problem. The Multiple Pass Floating Stope [25] algorithm was introduced to address issues with the Floating Stope algorithm. It generates additional output beyond the reserve area to help engineers determine the best possible scenarios. However, the need for additional assistance from engineers to determine optimum scenarios prevented this algorithm from reaching the global optimum. A similar approach to the Floating Stope algorithm was adopted in the Maximum Value Neighborhood (MVN) [26], which can enforce geomechanical constraints while selecting the best possible sets that yield the highest economic value. Nevertheless, this algorithm is sensitive to starting location factors that impact running time and the optimal result. One major advancement was introduced by Topal and Sens [27]. Their algorithms deliver 3D results that directly eliminate the overlapping stope problem in stope optimization. However, a major drawback is the selection of optimum stope sets in descending order, which automatically eliminates the possibility of better stope combinations. A different approach was proposed by Ba [28] with the Network Flow algorithm to identify the best stope layout via its centerline by linking the surrounding blocks. Geomechanical constraints are utilized to guide the algorithm, thereby determining the desired stope size. Its limitations in sublevel mining methods, which are only applicable to small mineralized bodies, create additional opportunities for further development. Sandanayake [18], [22] modified the Floating Stope algorithm by introducing several steps to eliminate drawbacks in the stope selection sequence and optimization starting point. Key optimization steps that address these issues include generating the stope value, eliminating negative-valued stopes, and selecting the best stope combinations that generate the maximum economic value. However, this algorithm's limitations include the fact that cost variables used in optimization do not consider mining size. Villalba and Kumral [21] proposed an algorithm to quantify the impact of internal dilution in stope optimization. This algorithm can enforce geomechanical constraints and use a linear penalty function to decrease the value of potential stopes below the cutoff grade. However, limitations remain in how the algorithm quantifies geological risks, such as grade fluctuations. Recent studies have shown a trend towards utilizing heuristic-type algorithms to solve stope optimization problems. Generating an optimal stope layout in 3D form for large-scale cases is becoming feasible. Heuristic algorithms show potential for solving extensive cases within a reasonable amount of time.

A common approach to managing large-scale cases in heuristic-type algorithms is to divide the problem into smaller sub-problems. The mining level can be considered a sub-object where stope optimization is performed. First, the best combination that generates the highest possible economic value is determined, followed by the best combination of stope sets. This approach significantly reduces computational resources, as optimization only occurs at each level. A recent study by Sari and Kumral [29], [30] introduced a heuristic approach that applies step-by-step optimization, where stope layout optimization is separated into two major parts: level determination and stope layout determination. This effectively decreases problem size and improves computational time. To impose geotechnical constraints, the algorithm uses static geotechnical parameters by specifying the maximum and minimum stope sizes along each optimized axis. Although the method enforces stope sizes to conform to rock conditions, a significant gap exists because it does not directly consider the variability and detailed characteristics of rock condition parameters. In some cases, a potentially larger stope in certain areas is forced to have a smaller geometry due to the conservative geometry typically chosen for a given area. Additionally, geological structure, water conditions, and stope orientation are key factors that influence stope design. Moreover, the method of imposing geostatistical conditions using a range of dimensions eliminates potential optimization from a geomechanical perspective, as stability analyses are run separately.

To achieve optimal conditions, it is crucial to consider all relevant parameters in the optimization objectives. This drives the need for studies that integrate optimization algorithms with stability methods. Although the demand for such integration is substantial, studies that incorporate stability analysis into stope optimization remain limited. Esmaili [31] successfully integrated the Mining Rock Mass Rating (MRMR) algorithm into the Network Flow algorithm. MRMR was implemented in the stope optimization algorithm by enforcing stope dimension recommendations for each mining level. In addition to RMR, geomechanics-based dimension approaches, such as the Mathews Stability Graph [9], are widely used in the mining industry. Danu [32] addressed this by integrating Mathews Stability Analysis into a stope optimization algorithm. The algorithm is capable of maximizing economic value while maintaining individual stope stability relative to the geomechanical variability introduced within the block model. However, level determination in the early phase of the algorithm remains a shortcoming. This study addresses the gap in earlier algorithms by applying the Mathews Stability Graph in optimization algorithms, where stope dimensions and levels were pre-determined by user input.

2. Methodology

2.1. General optimization approach

The optimization was carried out through several stages, as outlined in Figure 1. In the first stage, the block's economic value is calculated based on geological attributes and economic parameters. At this stage, the economic value of each block is calculated using Equation (1):

$$v_b = \left[\left(p - r \right) \cdot g \cdot y - \left(e + c \right) \right] \cdot t \,, \qquad (1)$$

where:

v_b – economic value, $;
p – metal price, $/gram;
g – metal grade, g/ton;
y – mining recovery, %;
e – processing cost, $/ton;
c – refining cost, $/ton;
t – mining tonnage, tons.

The next stage consists of two steps that run in parallel: optimal level determination and stope dimension determination using the Mathews Stability Graph. The optimal set level was determined by identifying the best combination of levels that yielded the highest economic value, while the stope dimensions were determined by applying Mathews Stability Graph analysis based on geotechnical data provided in the block model. The stope size recommendation is then represented by allowable mining blocks in three axes for each of the blocks. Geotechnical constraints were subsequently applied to the stope layout at each block location, ensuring that the stope layout met both practical and geomechanical considerations.

In the later stage, stope layout optimization, several steps are involved, including positive stope generation, optimum stope generation, and stope visualization. A positive stope is generated from blocks at levels predetermined in the earlier stage. While generating stopes, the algorithm adheres to geomechanical constraints imposed by the stope dimension recommendations.

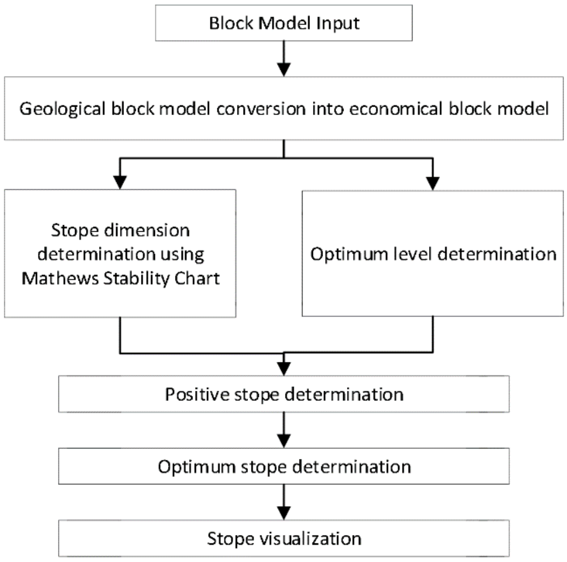

Figure 1. General optimization flow

Subsequently, the algorithm updates the stope's economic attributes while eliminating negative-valued stopes. These positive-valued stopes are then used for stope optimization. Heuristic optimization is employed to generate the best possible set of stopes that produce the highest economic value. Figure 1 provides a brief summary of the algorithmic steps proposed in this study.

2.2. Objective function

A mathematical formulation to maximize the economic value of the project was implemented in this study. A binary variable was used to differentiate between mined and unmined stopes. The optimal economic value was subsequently calculated by summing the economic values of the mined stopes. Equation (2) presents the mathematical formulation used in the objective function:

$$V = \sum i_{stope} \cdot v_{stope} \,, \quad s \in S \,, \qquad (2)$$

where:

V – total economic value, $;
v_{stope} – stope economic value, $;
i_{stope} – 1 for the mined stopes; otherwise, 0;
$s \in S$ – sets of stope, s.

2.3. Overlapping constraint for stopes

Overlapping stopes are common in stope optimization problems and have been highlighted in various studies [18]. This condition occurs when the generated stopes overlap with each other, leading to double counting of stope attributes. As a result, reserves are reported to be greater than they actually are, creating an overly optimistic value, particularly in terms of reserves and economic value. Various approaches have been implemented to eliminate this issue, some of which use a mined block identifier to prevent a block from being mined twice. Consequently, stope generation is prohibited in the vicinity of already mined blocks.

Figure 2 illustrates two stopes that overlap. A binary variable is used to mark overlapping blocks and prevent stopes from overlapping.

While generating a stope, blocks are tagged as being mined. Additionally, another process checks whether the blocks have already been mined.

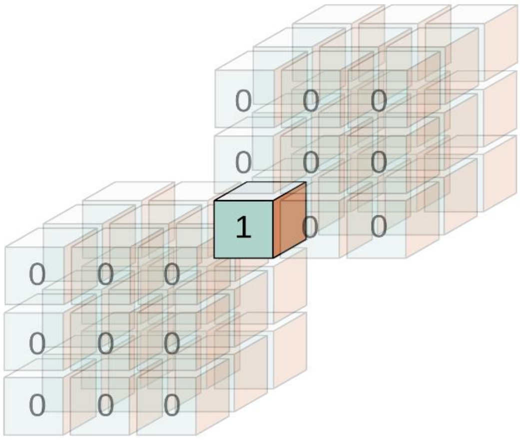

Figure 2. Overlapping stope constraint

If a block has already been mined, a value is assigned to the overlapping blocks. Equation (3) is then used to eliminate overlapping stopes by accumulating overlapping indexes for those stopes:

$$\sum i_{overlap} < 0, \quad b_{stope} \in B, \tag{3}$$

where:

$i_{overlap}$ – 1 for overlapping blocks; otherwise, 0;

$b_{stope} \in B$ – sets of blocks within stope, s.

2.4. Geomechanical constraint using the Mathews Stability Graph

To impose a dimensional constraint, practical considerations are addressed, and the Mathews Stability Graph [33] is applied within the optimization steps. Geomechanical data present as attributes in the block model are used in the formulation, including the Q′ value, factor A, factor B, and factor C. Geomechanical data must be available for every block, as calculations are performed for each block. Contrary to common practice, where stability analysis is conducted only in areas with significant differences in rock conditions, it is performed for every block location to account for the varying rock conditions characteristic of the optimization area.

The stability number N′ is required to determine wall stability in the Mathews Stability Graph. Thus, the N′ value for each block is essential, as the stability analysis is based on block location. To calculate this, N′ is derived using Equation (2), where Q′ is the value of Q′, and A, B, and C are the values of factor A, factor B, and factor C, respectively, in the Mathews Stability Graph:

$$N = Q' \cdot A \cdot B \cdot C. \tag{4}$$

Stope wall recommendations are imposed by applying an allowable hydraulic radius to the stope wall being analyzed. It is possible to generate a hydraulic radius for every potential stope wall at each location within the orebody. The hydraulic radius is determined from the N′ value, which was calculated at an earlier stage using Equation (5):

$$HR = 10^{0.573+0.388 \cdot \log N}. \tag{5}$$

The algorithm runs in 3 dimensions so that the determination of the *HR* value on each paired wall is determined by the smallest value indicator. Recommendations for the length and width at a certain iteration point are calculated based on Equations (6) and (7), where nys, nxs, nzs, HR_{1r}, HR_{3r}, Ho, and Lo are the number of blocks allowed on the y axis, the number of blocks allowed on the *x* axis, the number of blocks allowed on the *z* axis, the lowest hydraulic radius between the hanging wall and footwall, the lowest hydraulic radius between the back and front stope, the height of the block and the length of the block, respectively.

$$nys = \frac{2 \cdot HR_1 (nzs \cdot Ho)}{((nzs \cdot Ho) - 2 \cdot HR_1) / Lo}; \tag{6}$$

$$nxs = \frac{2 \cdot HR_3 (nzs \cdot Ho)}{((nzs \cdot Ho) - 2 \cdot HR_3) / Ho}. \tag{7}$$

Recommendations for dimensions can be made based on the number of allowable blocks, as the creation of stopes in the optimization algorithm relies on the multiplication of block dimensions.

2.5. Level determination

The level in the algorithm is determined by inputting a fixed stope height, ensuring that each mining level has the same height. The selection of the best combination of levels is then carried out in several stages, following Sari [34], as follows:
- identify the combination of level sets in the block model;
- sum the economic value of each mining level;
- sum the economic value of the total level set combinations;
- select the level set with the best economic value.

Figure 3 illustrates how the algorithm selects the best level, where all levels that meet the condition of being a multiple of the stope height are identified for subsequent grouping into several sets. Each set consists of levels that do not overlap with each other. Once these sets are identified, the economic value of each level is calculated. In this algorithm, the length of the level corresponds to the entire length of the strike orebody. The economic value of the level is obtained through Equation (8), where v_{block} is the economic value of each block, and tag_r is the ore marker index for a particular level:

$$v_{level} = \sum v_{block} \cdot tag_r \forall tag_r = 1. \tag{8}$$

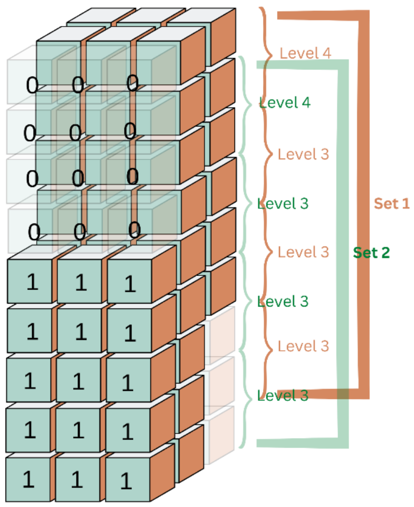

Figure 3. Set determination based on the non-overlaping level

The calculation of the economic value of each set is then carried out on levels that do not overlap with one another through Equation (9), where v_{level} represents the economic value of each level, while the value of tag_l indicates the numbering of each level that is included in a set:

$$v_{set} = \sum v_{level} \cdot tag_l \forall tag_l = 1. \quad (9)$$

The selection of the combination set is based on the economic value of each set, with the highest economic value serving as the basis for the final selection. The best set is chosen and used as the input for the next stage of optimization. Figure 4 illustrates the best combination of levels based on the economic value of each set.

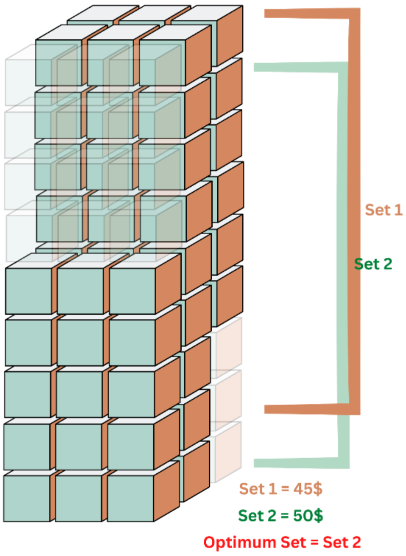

Figure 4. Optimum set determination based on economic value

2.6. Optimization

Optimization is performed using the heuristic algorithm proposed by Sandanayake et al. [22]. The iteration is carried out step by step by creating several sets, ultimately determining a unique set by merging sets that do not overlap with each other. Each of these sets is then saved, and the economic value of each unique set is compared. The unique set with the highest economic value is selected as the best solution from the optimization process.

3. Case studies

3.1. Data and parameters

The algorithm was tested using a block model that simulates the existing orebody conditions in Indonesia. The data were categorized into geological, economic, and geomechanical types. Geological data include gold grade attributes, while economic data encompass parameters used for economic calculations. Geomechanical data include the parameters Q′, factor A, factor B, and factor C, which form the basis for stability calculations in the proposed algorithm. The study area was divided into two zones: the ore and the host rock areas.

Figure 5 (upper-left) shows the geological data in the form of gold grade in the rock, which is used to calculate the metal product obtained when a block is mined, measured in grams per block. In the block model, gold grade varies with elevation, ranging from 18.83 to 2.50 g/ton. The gold grade at the top two block levels (102.5 and 100 m) is 0 g/ton, indicating weathering conditions commonly observed in the field. Additionally, the lowest block level also has a gold grade of 0 g/ton, reflecting the decreasing grade distribution at very deep levels, consistent with typical gold distribution patterns in Indonesia [35]. The geological data and extent of the block model are presented in Tables 1 and 2.

Table 1. Geological properties of the block model

Descriptions	Symbol	Value	Unit
Numbers of block	B	32.562	blok
Block dimension	$Lo \times Wo \times Zo$	2.5×2.5×2.5	meter
Block type	–	Uniform	–
Ore block density	d_{ore}	2.36	ton/m^3
Host rock block density	d_{rock}	2.66	ton/m^3
Gold grade variation	g	0 to 40.32	gr/ton

Table 2. Extent of the block model

Descriptions	Symbol	Value	Unit
Maximum block centroid	x_{max}	145.00	meter
	y_{max}	267.50	meter
	z_{max}	35.00	meter
Minimum block centroid	x_{min}	100.00	meter
	y_{min}	100.00	meter
	z_{min}	102.50	meter

Economic data include commodity price components and cost components, such as mining costs, processing costs, and beneficiation costs. These values are set deterministically and used in the calculation of block economic value, as conveyed through Equation (1). Table 3 details the economic parameters representing typical commodity prices and mining costs in Indonesia with the stoping method. The results of the economic calculation for each block are shown in Figure 5 (lower-left), where the distribution of block economic values varies proportionally with the gold grade distribution.

Table 3. Economic parameters

Descriptions	Symbol	Value	Unit
Metal price	p	54.80	\$/gram
Mining cost	e	35.80	\$/ton
Processing cost	c	1.62	\$/ton
Refining cost	r	3.89	\$/ton
Global recovery	y	80.00	%

3.2. Scenarios and algorithm performance assessments

Three scenarios, namely, the fixed stope dimension (scenarios 1 & 2) and the proposed algorithm (scenario 3), are used to evaluate the performance of the algorithm: fixed stope dimensions (scenarios 1 & 2) and the proposed algorithm (scenario 3). Scenario 1 involves a fixed stope dimension based on the maximum allowable dimension determined from stability analysis. This is calculated by using the maximum N′ value presented in the data in correlation with the maximum allowable hydraulic radius.

The maximum dimensions are then calculated based on this allowable hydraulic radius. Scenario 2 represents a fixed stope dimension based on the minimum operational considerations allowed. Scenario 3 features a variable stope design based on the proposed Mathews stability modules integrated within the level optimization algorithm. A summary of these parameters is presented in Table 4.

Table 4. Scenarios for algorithm trial

Scenario	Stope dimension ($l \times w \times h$)	Level constrained
Scenario 1	7.5×7.5×5	Unconstrained
Scenario 2	5×5×5	Unconstrained
Scenario 3	variable	Constrained

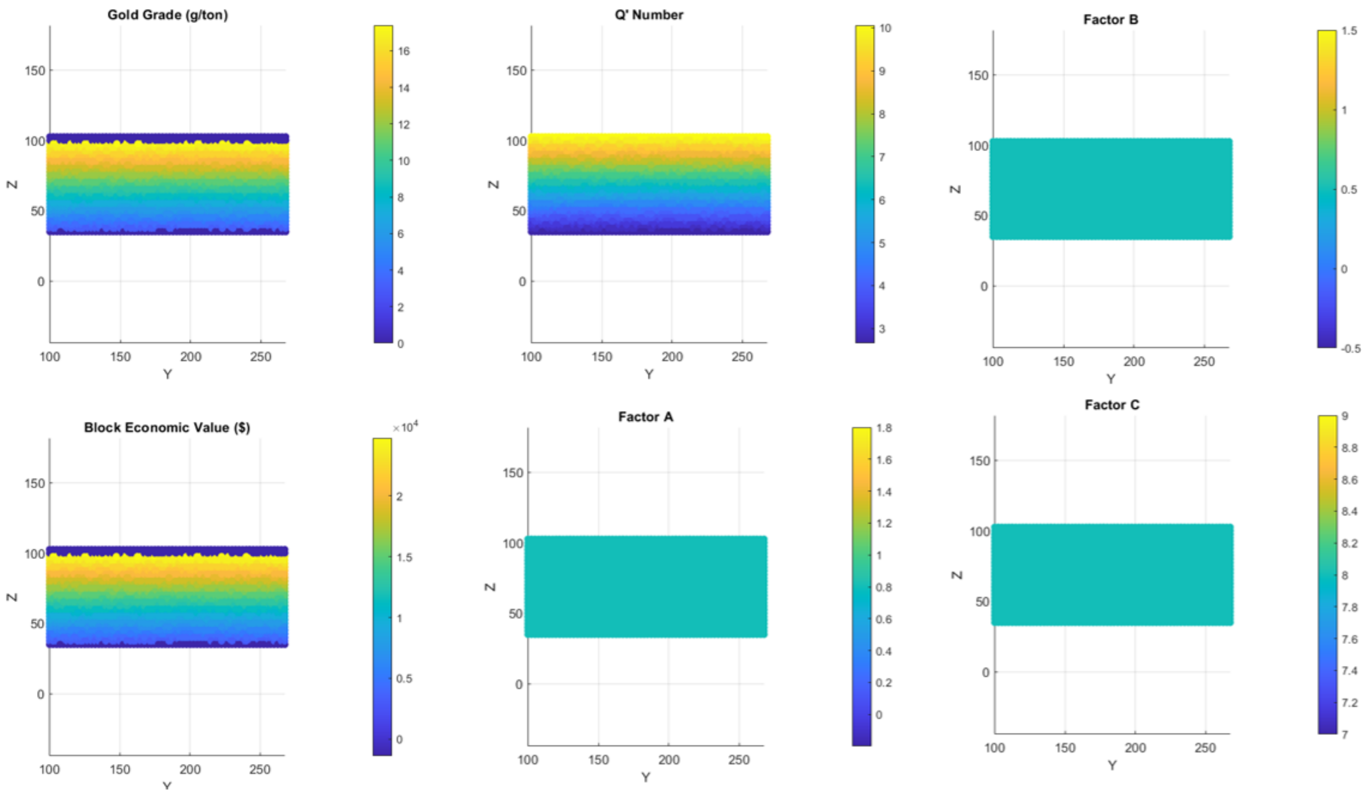

Figure 5. Block model data: gold grade (upper-left); block economic value (lower-left); Q' number (upper-middle); factor A (lower-middle); factor B (upper-right); factor C (lower-right)

A non-overlapping level constraint is used to analyze the impact of setting level constraints on stope optimization. This constraint is applied only in Scenario 3 and determines whether overlapping levels are permissible. By applying constraints on levels, the algorithm can create stopes on grouped levels, which enhances the optimization results to better meet practical criteria.

The results from each scenario are compared based on the generated level value, project value, and stope layout to evaluate performance. The level value represents the cumulative economic value of the project based on the specified optimal level set, without considering the stope layout. This economic value is derived solely from the cumulative block model value. A thorough examination is conducted on every potential set. The project value represents the cumulative value of the project achieved through the optimal stope layout, which is determined using optimization techniques. The values are compared across scenarios, as shown in Table 4. Additionally, the stope layout for each scenario is compared in Table 4 to assess the impact of the implemented non-overlapping levels.

The stability analysis module performance within the stope optimization methodologies is evaluated using back-analysis. This involves plotting the lowest stability number N′ on each wall against its hydraulic radius on a stability graph. Geomechanical property data (Q′, factor A, factor B, and factor C) are used to determine the N′ value, while the corresponding wall dimensions are used to determine the hydraulic values. Analysis is performed for every stope in the optimal set, and the Mathews graph is plotted based on the calculated values.

The feasibility of the algorithm is determined based on the stability of the generated stopes. If all stability plots fall above the stability line, the algorithm is considered successful.

4. Results and discussion

4.1. Optimum level

Table 5 shows the results of optimizing the best set of levels. The optimization process identified two sets (combinations) of mining levels. Of these, the first set has the highest economic value of $75.52 million, which is slightly lower than the second set with an economic value of $75.92 million. Consequently, the optimization in Scenario 3 begins with Set 1, positioned at an elevation of 35.

Table 5. Level optimum

Parameter	Level started, m	Economic value, $ millions
Set 1	35	75.52
Set 2	37.5	74.92

4.2. Optimum stope

Table 6 shows that the results are consistent with predictions that variations in stope dimensions will result in a more conservative number of stopes for scenarios with larger stope dimensions, as indicated by the acquisition of 265 stopes in scenario 3b compared to 797 stopes in scenario 2. Furthermore, this condition affects the smaller tonnage of the mined material, even though the metal content mined is greater than that in the other two scenarios. The variation in stope size applied to scenario 3 explains this phenomenon, where the algorithm has more flexibility in determining the optimum stope at various locations. Additionally, scenario 3 can be considered to have better operational performance than the other scenarios, as shown by the highest mining grade achieved, indicating that less waste material is mined while maintaining a stable shape.

Table 6. Optimization results

Parameter	Scenario 2	Scenario 1	Scenario 3
Number of stopes	797	197	265
Mined tonnage, thousand ton	241.4	240.67	195.87
Mined metal, kg	1939.59	1798.64	1837.14
Mined average grade, g/t	8.03	7.47	9.38
Economic value, mln $	69.96	67.25	67.49
Mining recovery, %	95.52	88.57	90.47

Conducting a stability analysis of the optimization algorithm can provide significant economic value while maintaining feasibility. This is evidenced by the economic value of the optimization results, which amounts to 67493300 and is positioned in the middle among the three scenarios.

Another aspect that can serve as an indicator of success in scenario 3 is the amount of mining recovery. In this scenario, a relatively good mining recovery is achieved, with a value of 90.47%, which is positioned in the middle among the three scenarios. However, further improvements are needed in the algorithm structure to ensure that integrating the stability analysis module does not reduce the economic results, as observed in scenario 3. Figure 6 shows that the application of the proposed algorithm yields non-overlapping level results, whereas scenarios 1 and 2 have overlapping level conditions. This overlapping condition does not meet the operational requirements of underground mining because material transportation is typically more efficient when mining is conducted in groups at the same level. In accordance with mining operational conditions, where stopes are mined at the same level, scenario 3 can be considered more representative of operational perspectives than the other two scenarios.

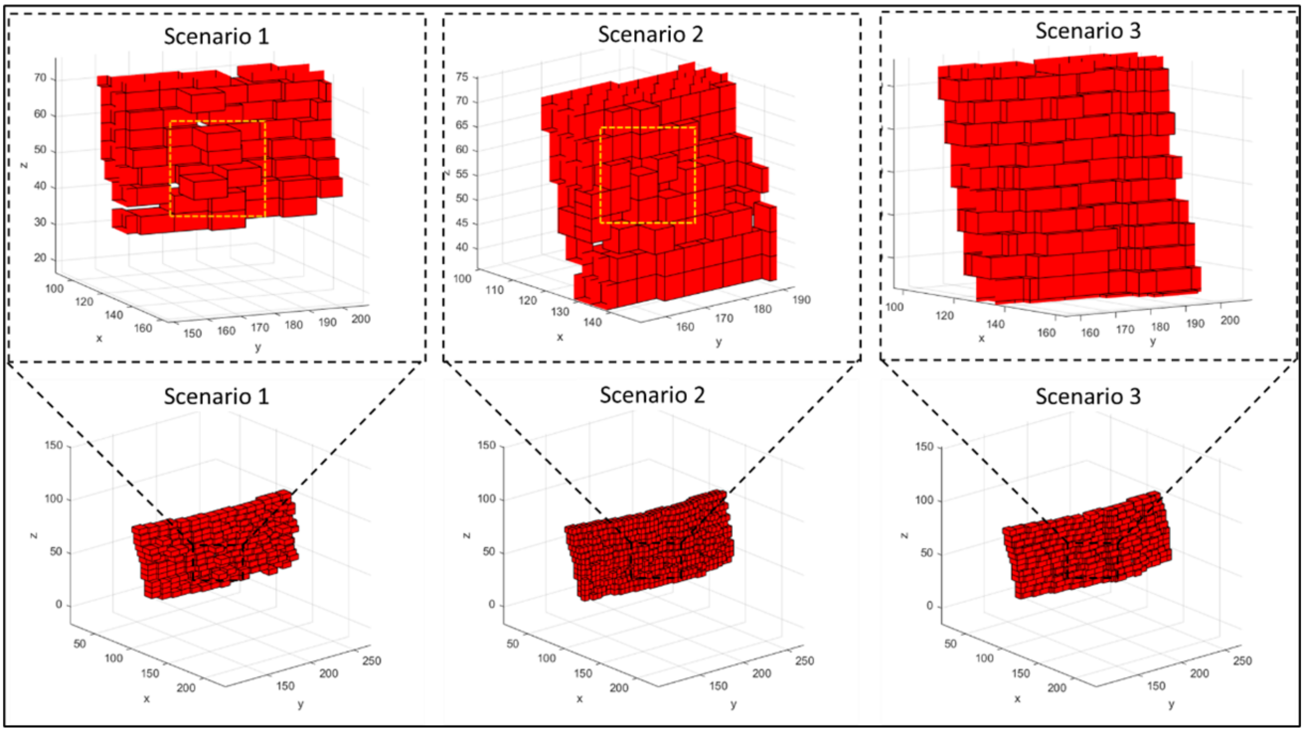

Figure 6. Optimized stopes: scenario 1 (left); scenario 2 (middle); scenario 3 (right)

4.3. Stope stability confirmation

Figure 7 shows the wall plot in the stability graph. There are four types of walls based on their position: the red, green, grey, and yellow points represent hanging walls, foot walls, front walls, and back walls, respectively. Variations in rock properties result in a wide spectrum of stability numbers, leading to a vertical distribution of the points, while a narrow distribution of the hydraulic radius indicates fewer options for the stope dimensions.

Due to the optimization framework, which assesses the stability of the stope walls in pairs, each stope's hanging wall and foot wall will have similar hydraulic radius values. All optimized stope walls are stable, as indicated by the population of points above the stability line. Furthermore, the excellent stope economic value and stable conditions at each stope wall demonstrate the satisfactory application of the Mathews stability analysis to the optimization method.

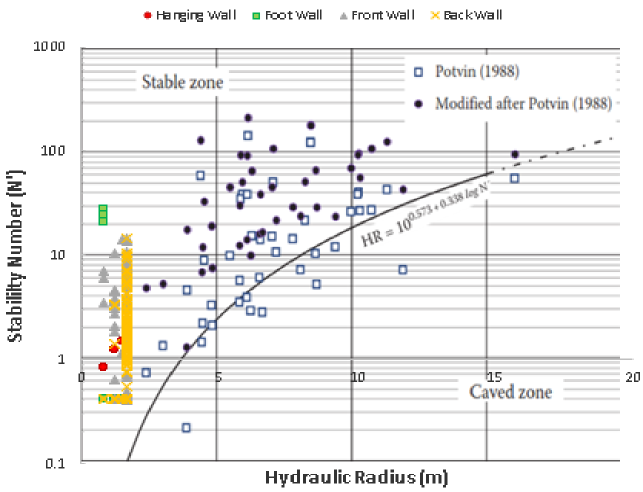

Figure 7. Stability confirmation for the hanging wall and footwall

4.4. Future perspectives

The delicate process of designing underground stopes involves numerous factors, particularly the economics of design and the properties of the rock, resulting in a complex problem that requires resolution. The common approach involves deriving design recommendations from an iterative stability analysis process and then undertaking underground design based on these recommendations. Additionally, as the mine progresses, the increasing underground rock data creates a lengthy process between stability analysis and stope design, leading to longer working hours, reduced productivity, and a lower probability of finding the optimal scenario. Stope optimization is one solution proposed in recent studies to address this issue.

Stope optimization considers many parameters during its process to achieve the best possible design. These parameters primarily focus on geology and economics, with the final valuation indicator being the dollar value. Most optimization methods provide a maximum and minimum range of input values for stope dimension parameters to meet geomechanical requirements. An iterative process is necessary to address the constant updating of geomechanical data.

This study addresses the gap between stability analysis and optimization in stope design, resulting in improved workflow and faster analysis of optimization scenarios. By applying the proposed algorithm, engineers can better focus on analyzing improved strategies due to the integration. At the end of the study, potential integration could involve combining current methodologies into strategic scheduling.

Scheduling of underground stope mining is also a primary challenge in underground optimization. Integrating current stope stability techniques with stope scheduling optimization will create a breakthrough, significantly aiding engineers in overcoming complex problems in underground mines.

5. Conclusions

The mining level and dimension are critical parameters in underground mining planning, particularly in the stope method. The initial determination of mining levels affects the location of subsequent mining levels and ultimately impacts the project's feasibility. This challenge is compounded by the complexity of determining the stope dimensions. A general algorithm is employed in mining planning to address this issue. One such algorithm is the mining level determination algorithm. However, existing algorithms often lack the integration of stability analysis techniques, which means the resulting design does not directly account for stope stability. On the other hand, stability analysis techniques have advanced, including the use of the Mathews Stability Graph to assess stope wall stability. This study aims to integrate the Mathews Stability Graph into an optimization algorithm for mining levels, allowing the existing algorithm to incorporate stability analysis in a series of steps. The Mathews Stability Graph of the unsupported stope serves as the basis for determining the stability of the stope walls in the algorithm.

The optimization results show that the optimized stope offers good economic value and mining recovery while maintaining stable dimensions. Additionally, the application of mining level constraints in the proposed algorithm demonstrates that the optimized stope is more practical. From a geomechanical perspective, a re-analysis of the optimized hanging wall, foot wall, front wall, and back wall revealed stable conditions, indicating that the algorithm successfully produced a stable stope wall.

In general, the integration of the Mathews Stability Graph into the algorithm effectively meets the study's goals, achieving optimized stope results that are stable while maintaining project feasibility. This algorithm can also be applied at the beginning of mine planning, especially during the stope optimization phase, as an initial indicator of mining feasibility. However, improvements are needed to maximize economic value and enhance the ability to interpret available geomechanical data in the field today.

Author contributions

Conceptualization: DP, BS; Data curation: DP; Formal analysis: DP, TK; Funding acquisition: TK, BS; Investigation: TK, MNH; Methodology: DP, BS; Project administration: TK, MNH; Resources: TK, BS; Software: DP; Supervision: TK, BS, MNH; Validation: DP, TK, MNH; Visualization: DP; Writing – original draft: DP, TK; Writing – review & editing: TK, BS, MNH. All authors have read and agreed to the published version of the manuscript.

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Conflicts of interests

The authors declare no conflict of interest.

Data availability statement

The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

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Інтеграція факторів стійкості A, B і C на графіку стійкості Метьюза до алгоритму оптимізації рівня видобутку корисних копалин

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Мета. Це дослідження спрямоване на вдосконалення підходу до алгоритму оптимізації рівня видобутку очисного вибою шляхом інтеграції аналізу стійкості з використанням графіка стійкості Метьюза.

Методика. Мова програмування використовується для інтеграції графіка стійкості Метьюза до алгоритму оптимізації рівня видобутку очисного вибою на попередньому етапі оптимізації, забезпечуючи розмірні обмеження на основі стану гірської породи. Обгрунтування алгоритму проводиться з використанням трьох сценаріїв, що відображають стан гірської породи в блоковій моделі: фіксовані розміри очисного вибою з максимальним розміром очисного вибою, фіксовані розміри очисного вибою з мінімальним розміром очисного вибою та змінні розміри очисного вибою на основі запропонованого алгоритму. Крім того, для перевірки стійкості очисного вибою в алгоритмі оптимізації, стійкість кожної стінки очисного вибою підтверджується шляхом побудови зворотного графіка на графіку стійкості.

Результати. Результати оптимізації показують, що оптимізований очисний забій забезпечує прийнятну економічну цінність та рівень вилучення при збереженні стабільних розмірів. Виявлено на основі повторного аналізу оптимізованого висячого боку, лежачого боку, передньої стінки та задньої стінки стабільні умови з геомеханічної точки зору, що вказує на те, що алгоритм успішно створив стійку стінку очисного вибою. Рекомендовано алгоритм також застосовувати на початковому етапі планування гірничих робіт, особливо на етапі оптимізації очисного вибою як початковий індикатор здійсненності видобутку.

Наукова новизна. Оптимальна конструкція очисного вибою часто визначається економічним параметром очисного вибою, тоді як геотехнічні змінні, легко доступні в блоковій моделі, ігноруються. Запропонований алгоритм спрямований на включення аналізу стійкості з використанням графіка стійкості Метьюза до алгоритму оптимізації рівня видобутку очисного вибою.

Практична значимість. Метод був успішно апробований із використанням даних блокової моделі, що імітує умови реального рудного тіла в Індонезії. Крім того, цей метод може використовуватись проєктувальниками гірничих робіт на ранній стадії техніко-економічного обґрунтування.

Ключові слова: *оптимізація очисного вибою, графік стійкості Метьюза, стійкість очисного вибою, планування очисного вибою, алгоритм*

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