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Modelling mineral reserve assessment using discrete kriging methods

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Abstract

Purpose. Develop an efficient assessment model for amber placers within the analyzed block involving its content in the samples taken inside the block as well as out of it.

Methods. To obtain results, a complex of scientific procedures has been applied consisting of system analysis; generalization; mathematical statistics; mathematical, perfect, and psychical modelling; field experiment; and forecasting.

Findings. Methods to assess amber placer reserves have been substantiated. The potential of discrete kriging has been involved inclusive of the search for the best evaluation of amber content within the analyzed block taking into consideration the mineral percentage in the samples taken inside the block as well as out of it.

Originality. The research has helped understood that the discrete kriging results help assess average amber content within the square and curvilinear zones tending to the central well (in terms of amber percentage) both in the central well and in wells of the first and second contact zones.

Practical implications. Identification of the most efficient technique, assessing amber reserves, improves forecasting reliability of the resources with minimal cost.

Keywords: *model, amber, efficiency, assessment, kriging, placer, well, mineral*

1. Introduction

No field can be developed efficiently without achievement of clear quantitative mapping of the mineral content within the deposit [1]-[3]. Mining procedure should be combined with the indices in terms of energy intensity; aggregate parameters; and degree of the rock mass differentiation [4], [5]. Under the circumstances, it becomes necessary to develop new methodological approaches for evaluation of the mineral reserves being the prerequisite and comparable with further mining.

Systematic and timely mineral resource accounting is extremely important in Ukraine [6]-[8]. According to the Subsoil Legislation, despite type of ownership, all the organizations, engaged in geological studies, should provide reliable geological structure of bowels and determination of both quantity and quality of the main and associated minerals as well as their useful components [9]. Accurate assessment of the quality and spatial conditions of the resource occurrence favours rational subsoil use and protection [10]-[13].

The prospected and previously estimated mineral resources are calculated within the established frameworks [14]. Construction of the deposit frameworks should involve maximally geological data determining regularities of spatial occurrence of the minerals within the earth's crust structures, and the mining procedure demands to ensure rational subsoil use as well as economic feasibility of the field development [15]-[18].

Resource contouring in the bowels is to identify the general industrial framework separating the mineral from enclosing rocks. The reserves are distributed based upon determination of block sites within the general contour, differing in their structure, morphology, prospecting degree, and the mineral composition. As opposed to calculation of the prospected and previously assessed reserves, the prospective subsoil resources are not geometrized [19], [20].

All the techniques to calculate reserves are the two basic operations: mapping of complex in shape mineral accumulations into the equal in volume but simpler in shape geological bodies [21]-[23]; and calculation procedures providing reliable extrapolation of geological prospecting parameters obtained in the certain exploration cross-sections to the subsoil volume neighbouring them [24], [25].

Construction of frameworks, limiting the prospected bowels, involves maximally the geological data determining regularities of spatial mineral occurrence within the specific earth's crust structures [26], [27]. Moreover, the contouring of commercial mineralization takes into consideration mining procedure demands for mining economy expressed by means of condition parameters.

If clear natural edges between minerals and enclosing rocks are not available, commercially valuable sites should be contoured depending upon some specified (boundary) value of the useful component [28]. Boundary value is such a minimal content of a useful component in terms of which the sample may be included to a productive contour [29]-[32]. Apparently, the idea of geometry and structure of productive

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deposits as well as average percentage of useful components within them depends upon the boundary value [33], [34]. The lower boundary value is, the greater geometry of the deposits are, the simpler their shape is, and the lower mineral content is. The criterion to select the boundary value should become a degree of spatial occurrence of the useful components within the mineralized subsoil correspondence to the engineering conditions providing maximal mining efficiency [35]-[38].

If in the context of such a boundary value, being equal to minimally commercial, numerous small and isolated accumulations of mineral are contoured then it is expedient to lower the boundary value down to a boundary in terms of which the majority of such accumulations is united by a unique contour. Such an approach will be expedient for amber placers associated with Rivne Region bowels [39]-[41].

Optimal boundary value is that one under which maximum subsoil use is provided as well as high mining efficiency [42], [43]. The available methods substantiating conditions is not perfect since it cannot ensure obvious decision while selecting optimal mining procedure due to lack of the unified criterion to assess a deposit. Use of numerous interrelated conditioning criteria results in many alternatives, and excludes the possibility of their searching. In addition, the basic reserve contouring indicators include their insufficiently specified engineering and economic substance which complicates frequently quantitative and qualitative assessment of the reserve mining [44]-[47].

Volumes of productive zones, deposits, or blocks; volume weight of the mineral; and useful component content are the output geological prospecting calculation parameters [48]. The first two parameters are obvious since substance is in their names. Consider the third parameter.

Content of useful components within the prospected cross-sections is identified according to the analysis of placer samples [49], [50]. If content of useful components, identified owing to the prospected cross-sections, extends to neighbouring subsoil then errors of their assessment occur [51]-[53]. Linear interpolation errors results from the fact that spatial content variability inherits more complex laws. Moreover, linear interpolation errors are imposed by errors arising due to unbalance between average useful component content within the samples and its average content within the assessed blocks. Generally, samples with low content result in the underestimation; in turn, samples with high content result in average content overestimation within the evaluated subsoil volume [54]-[58].

Exploration grid geometry as well as the assessed subsoil volume geometry influences the interpolation error value [59]. Determination of the interpolation errors should involve complex calculation procedures; nevertheless, they can be represented digitally with an estimate of probability that the calculated average will stay within stable limits. The inconsistency between average content in samples and in the assessed volumes decreases naturally along with the increase in sample dimensions.

As opposed to interpolation errors, always having final values, extrapolation content errors may achieve abnormally high values; the matter is that empiric data expansion beyond the prospected volume never excludes the possibility of their practically complete non-confirmation. Standard statistical and non-statistical procedures are of little use to extrapolate content in the context of geological prospecting cross-sections outside the explored volume boundaries. In this case, it is required to apply geometrization methods, which take into consideration characteristics of nonrandom variability of geological prospecting parameters, i.e. trend analysis.

Average block content is defined as average weighted according to actually calculated layer thickness or using weight functions as for mineral structure anisotropy; exploration grid geometry; and the block being calculated.

2. Methods

Selection of a weight function to obtain the least dispersion for an average assessment is a rather complex mathematical problem having no general solution. As for the specific exploration conditions of many ore deposits, it was solved by G. Matheron [60] and called kriging. Kriging is a search for the best estimation of a mineral content within the calculated block taking into consideration its percentage in samples taken both inside the block and out of it. The approach should be considered as the most feasible for amber deposits with the use of sequence in exploration, design, commercial testing, and development [61]-[63].

Sense of kriging is as follows. Each sample content is labeled by such a weight to provide minimal dispersion of an average percentage assessment. Weight of samples may be defined using geostatical methods involving geometry, dimensions, and mutual placing of samples and the block being evaluated. Generally, the farther the sample from the block centre is, the lesser weight it is labelled:

$$Z^* = \sum_{i=1}^{n} a_i x_i \ , \qquad (1)$$

where symbols are represented in the kriging problem interpretation for a geometrically homogeneous field being reduced to identification of the best linear evaluation of actual average Z content within a block according to a number of samples with x_1, x_2 …x_n content inside the assessed block and out of it where weight coefficients are a_1, a_2…a_n. The two conditions should be fulfilled:

– truth Z content and its Z^* assessment should have similar average value throughout the geometric field, i.e. average value of Z-Z^* error should be equal to zero;

– a kriging coefficients should have such values in terms of which evaluation dispersion of the truth $D(Z\text{-}Z^*)$ content would assume its minimal value being recorded in the form of equation systems to be linear relative to a_i.

Discrete kriging helps obtain the average content assessment within a square zone. Moreover, discrete kriging use specifies substantially evaluation of the average-block content of useful components. The key practical kriging relevance is the possibility to avoid systematic errors in average content values as for rich blocks [64]-[66].

The problem of a weight function selection is closely connected with the outstanding sample problem arising frequently while prospecting amber placers in Rivne Region [67], [68]. Outstanding samples are those ones with abnormally high content where useful component percentage is much higher in random sampling to compare with similar useful component content within the calculated volumes.

The idea to minimize evaluation dispersion of a useful component average content in blocks with outstanding samples may become the basis for outstanding sample identification and consideration. For the purpose, one should know geometry of actual zones where the outstanding samples are

influential; and take into account the geometry impact as well as the number of the certain samples [69], [70]. However, such procedures, identifying and considering outstanding samples, are not available.

In the first approximation, the problem can be solved with the help of kriging.

Based upon the abovementioned, detalization of the research, intended to improve the efficiency of amber mining, should involve the following [71]-[73]:

– mathematical and computer modelling to assess the mineral reserves through discrete kriging methods;

– mathematical and computer modelling to assess the mineral reserves through continuous kriging methods;

– reassessment of the amber reserves based upon the mathematical modelling towards expansion of deposit contours with minimal commercial and boundary amber content.

To assess accurately amber reserves within the deposits having mainly curvilinear boundaries, kriging theory is added by one of mathematical modelling technique, i.e. finite difference method with the use of numerical conform mapping of flat curvilinear areas onto a parametrical rectangle. The idea relies upon transformation of an output boundary-value problem to new independent variables ζ and η of G_ζ area; complex potential of the parametrical rectangle; and the transformed problem solving within it.

In this context, numerical conform mapping of G_z area onto the parametrical rectangle should be constructed first; the abovementioned is considered as a separate problem. In this case, orthogonal difference grids have been applied inside the area with floating nodes along a physical profile of the deposit periphery.

Consider two algorithms to construct the numerical conform mapping. Algorithm one is based upon numerical solving of two sequences of Dirichlet problem for Laplace equations as well as upon alternation of the internal and external iteration processes to specify coordinates of internal nodes of a difference grid and floating nodes along the area boundary. To improve the algorithm reliability in the case of areas with irregular boundaries, it is convenient to use arrangements with the fixed position of boundary nodes in the vicinity of angular points of the areas. Algorithm two relies upon the use of the nodes and boundary nodes as floating ones.

Figures 1 and 2 show horizontal G cross-section of the deposit, and extraction chambers placed within it. Figure 1 demonstrates square deposit where extraction chambers are arranged in a checkerboard pattern; and Figure 2 is a deposit limited by a random curve in the form of a simply connected region. Resulting from the survey works, some points $1, 2 \ldots n$ of the sampled deposit have been coordinated as $M(x_i, y_j, z_k)$ with the known amber content ρ_{ij}^k throughout the chamber depth. The task is to assess amber percentage within the deposit.

The available procedure substantiating amber conditions is not perfect since it cannot provide unambiguous solution while selecting optimal alternative due to the fact that there is no single criterion of amber deposit evaluation. Numerous interrelated conditioning indicators exclude the possibility of their complete search. Moreover, the basic conditioning indicators, concerning contouring of reserves, do not involve sufficiently specified engineering and economic sense; in such a way, qualitative and quantitative assessment of the mineral extraction turns out to be complicated.

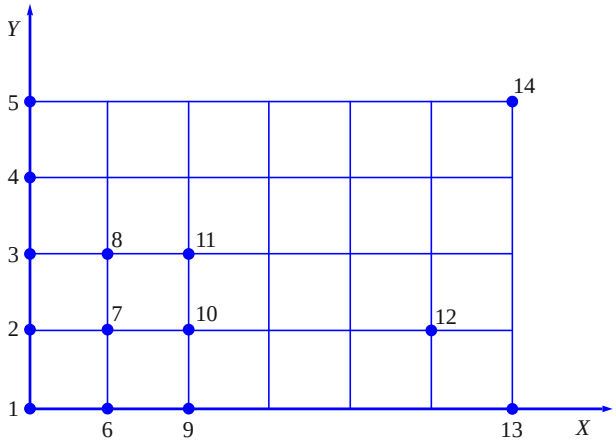

Figure 1. Rectangular field area: 1-14 are extraction chambers

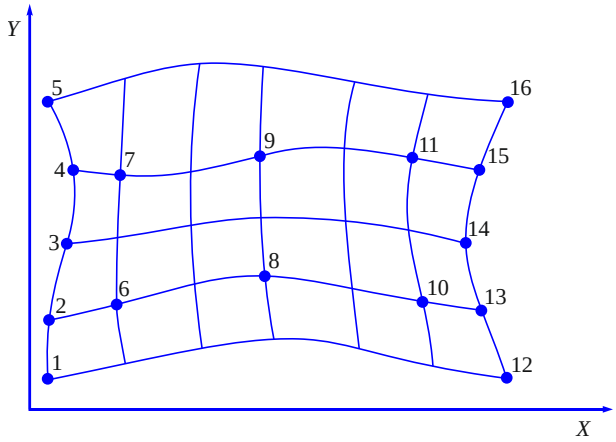

Figure 2. Physical area of a rectangular field: 1-16 are extraction chambers

Hydromechanical complexes for amber mining make it possible to take samples complying with the demands of reliability and the certified mapping. The abovementioned has defined the task aimed at reestimation of reserves in amber placers towards their increase and expansion of minimal commercial and boundary content of the mineral.

3. Results and discussion

3.1. Algorithm for the problem solving if the deposit is of a rectangular shape

Assess amber content within the deposit (which visible top portion is shown in Figure 1) using discrete kriging. First, evaluate the amber percentage within the separated block; then continue with determination of the best assessment of the separated block content applying searching results both inside the analyzed block and out of it. According to the methods, the results are involved by a weighing to provide minimum of the average value dispersion. In this context, kriging is implemented under discrete system of mineral deposit exploration.

Discrete (point) kriging is applied to interpolate the desired data into the specified mineral deposit points. Expediency to do that arises both at the stage of mineral occurrence identification and at operation stage when it becomes necessary to solve selective mining problems.

If during prospecting works a deposit is divided into chambers in the form of a square grid with $m \times n$ geometry, then kriging is used to identify weighs which should give

characteristic value within the central K chamber; within neighbouring L_1, L_2, L_3, and L_4 chambers; and within M_1, M_2, M_3, and M_4 chambers of a framing zone to determine the best assessment of an average characteristic value in the area of chamber K influence. In doing so, it is quite sufficient to involve only two contact zones neighbouring the block being calculated (i.e. K chamber influence area since in the majority of cases, use of data on more distant chambers cannot result in visible specification of the assessment.

Let amber content be known in ρ_{ij}^{k} chambers at some elevation H above ground. In this context, i and j indices are assumed as horizontal coordinates; and k index is assumed as a vertical coordinate. Moreover, amber content ρ_{ij}^{k} is known within the central chamber of the evaluated block. Hence:

$$V = \frac{1}{4}\left(\rho_{i-1,j}^{k} + \rho_{i+1,j}^{k} + \rho_{i,j-1}^{k} + \rho_{i,j+1}^{k}\right), \quad (2)$$

where:

V – average amber percentage within all the four chambers of a contact zone L extraction which includes L_1, L_2, L_3, and L_4 points where wells are arranged;

$$W = \frac{1}{4}\left(\rho_{i-1,j-1}^{k} + \rho_{i+1-1,j}^{k} + \rho_{i-1,j+1}^{k} + \rho_{i+1,j+1}^{k}\right), \quad (3)$$

where:

W – average amber percentage within all the four chambers of a contact zone M extraction which includes M_1, M_2, M_3, and M_4 points where the chambers are placed.

Average content of a useful component within the assessed block is defined by means of the Formula:

$$p = (1-\lambda-\mu)\cdot u + \lambda\cdot v + \mu\cdot W. \quad (4)$$

A symptotic formulas to define and μ kriging coefficients (4) under De Wijs model if both central and all wells in chambers of two neighbouring contact zones L and M in terms of small t values are as follows:

$$\lambda = \frac{(0.4277-\ln t)(0.5173-0.25\ln t)}{0.9121-1.4739\ln t+9/16\ln^2 t}; \quad (5)$$

$$\mu = \frac{(0.4277-\ln t)(0.0841-0.25\ln t)}{0.9121-1.4739\ln t+9/16\ln^2 t}, \quad (6)$$

if t values are large, then:

$$\lambda = 0.407;\ \mu = 0.017. \quad (7)$$

If so, then criging dispersion in the case one is:

$$\frac{1}{3\alpha}\sigma_k^2 = 0.1777 - \ln t - \left((\lambda-\mu)(0.4277-\ln t)\right), \quad (8)$$

in the case two it is as follows:

$\frac{1}{3\alpha}\sigma_k^2 = 0.311\frac{1}{t}$. In this context:

$$t = \frac{T}{h}, \quad (9)$$

where:

T – a mineral seam thickness;

h – a grid pitch.

Consequently, the total reserves of the field are identified through the Formula:

$$\sum_{k=1}^{n_1}\sum_{l=1}^{m_1} p_{2k,2l}, \quad (10)$$

where:

$n_1 = [n/2],\ m_1 = [m/2]$.

In such a way, the problem to assess amber reserves, disseminated dispersedly over the rectangular field, where the specified extraction chambers are arranged in the staggered manner, is formulated by means of Formula (10) and (3)-(6) are taken into consideration. Consider case two of the reserve estimation problem statement for a visible deposit shape in the physical area shown in Figure 3.

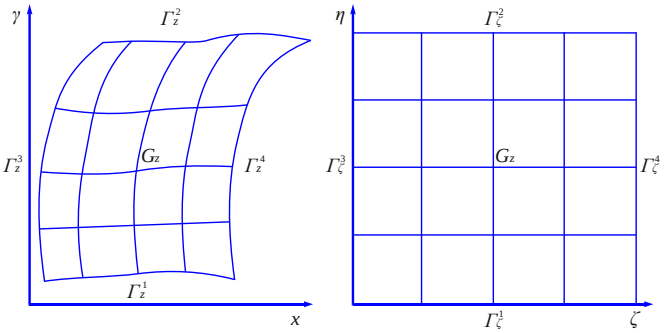

Figure 3. Physical area of G_z deposit with Γ^i_z boundaries and parametrical rectangle of complex potential G_ξ with Γ^i_ξ boundaries corresponding to it

To formalize the abovementioned problem statements for curvilinear outlining of the deposit framework, consider its horizontal cross-section as a simply connected area G_z limited by a piecewise smooth Jordan curve within (x, y) plane. The area boundaries may be preset by equations explicitly:

$$y = f_i(x) = \overline{1,k}; \quad (11)$$

implicitly:

$$g_i(x,y) = 0(x,y) \in \Gamma_z^i, i = \overline{1,k}; \quad (12)$$

and parametrically:

$$x = \phi_i(t), y = \psi_i(t), i = \overline{1,k}, \quad (13)$$

Moreover, boundary of the G_n area may be approximated through a rather dense number of points:

$$\begin{aligned}
&\left\{(x_{i0}, y_{i0}), i = \overline{0,n}\right\}(x_{i0}, y_{i0}) \in \Gamma_z^1;\\
&\left\{(x_{im}, y_{m0}), i = \overline{0,n}\right\}(x_{im}, y_{im}) \in \Gamma_z^2;\\
&\left\{(x_{oj}, y_{0j}), i = \overline{0,m}\right\}(x_{oj}, y_{0j}) \in \Gamma_z^3;\\
&\left\{(x_{nj}, y_{nj}), i = \overline{0,m}\right\}(x_{nj}, y_{nj}) \in \Gamma_z^4.
\end{aligned} \quad (14)$$

As a result of the exploration, amber content distribution ρ_{ij}^{k} has been obtained for each design extraction chamber throughout its depth. We will be engaged in the methods distributing wells within physical area of G_z deposit, i.e. assignment of (x_{ij}, y_{ij}) coordinates.

To assess amber reserves in such a way, apply one of mathematical modelling methods, i.e. finite difference method with the use of numerical conform mapping. In this connection, consider a problem of numerical conform mapping of simply connected area G_z onto parametrical rectangle G_ξ of (ξ, η) plane (Fig. 3).

Let Γ_z boundary of G_z area be the closed piecewise smooth Jordan curve with three preset *A*, *B*, and *D* points passing while mapping into A_1, B_1, and D_1 points of a parametrical rectangle. Since such a mapping is a single one, point *C* is determined from its correspondence to a vertex C_1 of the parametrical rectangle from the condition of floating along the area contour.

After the numerical conform mapping is over and conform difference grid is obtained, coordination of chamber is achieved within the grid nodes.

Further, the content of useful components in the deposit, limited by a curvilinear contour, is assessed through a kriging method applied similarly to herein before.

Assume that it is required to map conformally curvilinear square area of G_z deposit of (*x*, *y*) plane onto parametrical rectangle G_ξ of (ξ, η) plane (Fig. 3). Denote upside and downside of the curvilinear G_z rectangular using Γ^1_z and Γ^2_z; and and its left and right sides using Γ^3_z and Γ^4_z. Denote sides of the parametrical rectangle G_ξ as $\Gamma^1_\xi - \Gamma^4_\xi$ respectively.

G_z rectangle can be mapped conformally onto parametrical rectangle $G_\xi = \{0 \le \xi \le a,\ 0 \le \eta \le b\}$ with corresponding ratio of sides $M = a/b$ called as rectangle module. A_ξ, B_ξ, C_ξ, and D_ξ vertices of G_ξ rectangle correspond to four fixed A_z, B_z, C_z, and D_z points of G_z area. Mapping helps establish correspondence between the boundary points of G_ξ rectangular and G_z area as well as between the internal points of the areas.

Since angles are preserved during the conform mapping, orthogonal grid of two line groups may become a prototype of orthogonal grid. Such orthogonal grids are convenient; sometimes, they are necessary for certain numerical means applied to solve boundary problems on grids. Hence, it is expedient to develop orthogonal conform difference grids.

If one considers direct conform mapping of G_z area onto parametrical G_ξ rectangle, then it will be understood that isometric grid, which will cover G_z and consists of two mutually orthogonal line groups, transforms into orthogonal grid of G_ξ rectangle. Such grid lines will be in G_z, which will pass into isolines:

$$\begin{aligned} &\xi_i = i \cdot h_1,\ i = \overline{1, n-1};\\ &\eta_i = j \cdot h_2,\ i = \overline{1, m-1}, \end{aligned} \quad , \tag{15}$$

where:

$h_1 = const$ and $h_2 = const$ are grid pitches in G_ξ.

On the contrary, having selected orthogonal and uniform coordinate grid with h_1 and h_2 pitches (using two coordinates) in G_ξ rectangle, we will search for prototypes of coordinate lines within curvilinear rectangle G_z.

Consequently, the problem of numerical computation of conform parametrical rectangle G_ξ mapping onto curvilinear rectangle G_z is calculation of:

– *M* module of G_z area;

– coordinates of internal nodes $x(\xi_i, \eta_j)$ and $y(\xi_i, \eta_j)$ of a difference grid;

– coordinates of floating nodes $x(\xi_0, \eta_j)$ and $y(\xi_0, \eta_j)$ of Γ_z boundary of G_z area.

Consider different alternatives to set the problem on numerical conform parametrical rectangle G_ξ mapping onto curvilinear rectangle G_z. Each of the alternatives is intended to construct a difference grid within the curvilinear rectangle G_z for conform mapping of G_ξ rectangle onto G_z one under corresponding assignment of area G_z boundary and square or rectangular grid within G_ξ.

3.2. Alternative one

Γ_z boundary of G_z area is the closed piecewise smooth Jordan curve with the three preset points A_z, B_z, and D_z passing into A_ξ, B_ξ, and D_ξ points of parametrical rectangle G_ξ during mapping. Since the mapping is unique, then C_z point is found from correspondence to C_z vertex as a floating vertex of the rectangular G_z area. Depending upon the selected density of a coordinate grid as well as control of C_z point, following division of sides are applied in the parametrical rectangle:

$$A_\xi B_\xi = n \cdot h_1,\ A_\xi B_\xi = m \cdot h_2, \tag{16}$$

where:

h_1 and h_2 – a grid pitches.

Consequently, numerical constriction of the conform mapping with the help of difference grid for a simply connected area should involve random fixation of three boundary points corresponding to three vertices of the rectangle and selection of G_z area module, i.e. $M = n/m$. The grid has to be orthogonal within G_z area.

3.3. Alternative two

Γ boundary of G_z area consists of four piecewise smooth Jordan curves preset through the equations:

$$g_i(x, y) = 0 (x, y) \in \Gamma^i_z,\ i = \overline{1,4}. \tag{17}$$

Joined ends of the curves will produce four fixed points being A_z, B_z, C_z, and D_z. To solve the problem within the curvilinear rectangle, it is necessary to define *a/b* ratio of parametrical rectangle G_ξ, to enable availability of the required mapping with the rectangle vertices correspondence to A_z, B_z, C_z, and D_z points.

The represented statement for curvilinear rectangle, nodes are assumed as floating along the whole contour of G_z area boundary. The area module is defined by means of the problem statement; the unknown is defined during the mapping. The statement is to construct a difference conform grid within any finite simply connected G_z area limited by a piecewise smooth Γ divided randomly into four segments.

3.4. Alternative three

$\Gamma = \Gamma^1_z \cup \Gamma^2_z \cup \Gamma^3_z \cup \Gamma^4_z$ boundary of G_z area is approximated through rather dense table of $\Gamma = U^4_{i=1}\Gamma^i_z$ points:

$$\begin{aligned} &\left\{\left(x_{i0}, y_{i0}\right), i = \overline{0, n}\right\} \text{ for } \Gamma^1_z;\\ &\left\{\left(x_{im}, y_{im}\right), i = \overline{0, n}\right\} \text{ for } \Gamma^2_z;\\ &\left\{\left(x_{0j}, y_{0j}\right), i = \overline{0, m}\right\} \text{ for } \Gamma^3_z;\\ &\left\{\left(x_{nj}, y_{nj}\right), i = \overline{0, m}\right\} \text{ for } \Gamma^4_z. \end{aligned} \tag{18}$$

The conform mapping of parametrical rectangle G_ξ onto G_z area with the specified four points A_z, B_z, C_z, and D_z in terms of the preset *M* module, and four vertices is to be found.

According to the Riemannian theorem, concerning the specified simply connected areas, the conform mapping uniqueness is provided by correspondence of three boundary points of image and prototype. Solving the problem on conform mapping of rectangular area G_z on the parametrical rectangle G_ξ with correspondence of four points, being A_z, B_z, C_z, and D_z, to the rectangle G_ξ vertices is not always available. Such a mapping exists if only modules of the areas coin-

cide, i.e. when $M = a/b$ or if a square coordinate grid in G_ξ is selected, i.e. when $M = n/m$.

In the case one, the solution is unique. Hence, the grid of curvilinear coordinates in G_z can be influenced by selection of A_z, B_z, C_z, and D_z points; a/b ratio; and the grid density while selecting numerical values of natural numbers n, and m. In this context, a/b ratio selection will identify point C_z location within Γ boundary. Internal grid nodes are defined relying upon the solution algorithm. Boundary grid nodes are identified using proper algorithms for one of two coordinates; and coordinate two is determined using conditions of floating along Γ_z contour.

Case two involves the specific selection of four points of Γ_z boundary. The matter is that in the context of many boundary problems, G_z boundary area is of a special physical content where ${\Gamma^1}_z$ and ${\Gamma^2}_z$ curves are consumable lines; in turn, ${\Gamma^3}_z$ and ${\Gamma^4}_z$ are potential lines (or vice versa) of some field.

Solution algorithm should denote approximate identification of a/b ratio in such a way to make a floating node tend to C_z point. Hence, availability and uniqueness of the problem solving in terms of such a statement result from approximation algorithm of the problem solutions in the context of the first statement.

Consider the problem of conform G_z area mapping onto the parametrical rectangle G_ξ in the context of the represented statements. Let us suppose that such a mapping is implemented through the function:

$$\xi(z) = \xi(x, y) + \eta(x, y). \tag{19}$$

Below, you can find two mathematical models implementing the conform mapping.

3.5. Mathematical model 1 in its direct statement

It is common knowledge that under conform mapping, point coordinates of G_z and G_ξ areas are connected through Cauchy Riemann Equations:

$$\frac{\partial\xi}{\partial x} = \frac{\partial\eta}{\partial y},\ \frac{\partial\xi}{\partial y} = -\frac{\partial\eta}{\partial x}. \tag{20}$$

On sides of the parametrical rectangle G_ξ, $\xi(x, y)$ and $\eta(x, y)$ functions fulfil following Conditions:

$$\begin{aligned} &\xi(x,y)\Big|_{\Gamma_z^3} = 0, \xi(x,y)\Big|_{\Gamma_z^4} = a, 0 \le \eta \le b; \\ &\eta(x,y)\Big|_{\Gamma_z^1} = 0, \eta(x,y)\Big|_{\Gamma_z^2} = b, 0 \le \xi \le a. \end{aligned} \tag{21}$$

Moreover, the boundary conditions should be added by connection equation of border points of G_z area with near-border ones (Fig. 4):

– in terms of a form of Cauchy Riemann Equation:

$$\frac{\partial\xi}{\partial x} = \frac{\partial\eta}{\partial y},\ \frac{\partial\xi}{\partial y} = -\frac{\partial\eta}{\partial x},\ (x, y) \in \partial G_z; \tag{22}$$

– conditions of orthogonal lines of G_z area grid compliance with its boundaries not applying the grid Equations:

$$grad\ \xi \cdot grad\ \eta = 0, \text{ or } \frac{\partial\xi}{\partial x}\cdot\frac{\partial\eta}{\partial x} + \frac{\partial\xi}{\partial y}\cdot\frac{\partial\eta}{\partial y} = 0; \tag{23}$$

those ones applying the boundary Equations:

$$\begin{aligned} &grad\ g_i \cdot grad\ \xi = 0,\ for\ \Gamma_z^i, i = 1, 2; \\ &grad\ g_i \cdot grad\ \eta = 0,\ for\ \Gamma_z^i, i = 3, 4. \end{aligned} \tag{24}$$

In the expanded form they are as follows:

$$\frac{\partial g_i}{\partial x}\cdot\frac{\partial y}{\partial\eta} - \frac{\partial g_i}{\partial y}\cdot\frac{\partial x}{\partial\eta} = 0, (x, y) \in \Gamma_z^i,\ i = 1, 2; \tag{25}$$

$$\frac{\partial g_i}{\partial x}\cdot\frac{\partial y}{\partial\xi} - \frac{\partial g_i}{\partial y}\cdot\frac{\partial x}{\partial\xi} = 0, (x, y) \in \Gamma_z^i,\ i = 3, 4. \tag{26}$$

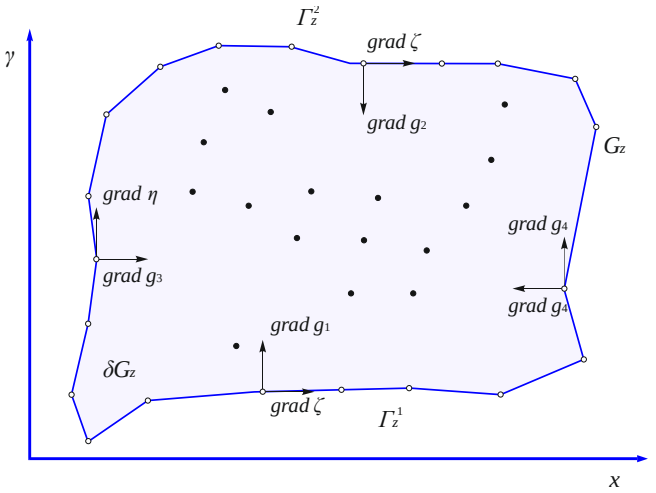

Figure 4. Near-border points of G_z area

3.6. Mathematical model 1 of the conform mapping problem in its inverse statement

We will search for the inverse conform mapping of G_ξ rectangle onto a curvilinear rectangle G_z being specified using the Functions:

$$x = x(\xi, \eta),\ y = y(\xi, \eta). \tag{27}$$

It is known that such an inverse mapping is represented through analytical function $z = z(\zeta)$ $z = z(\xi)$while also being conform, i.e. Functions (27) are also connected by means of Cauchy Riemann Equations.

Within segments of $\Gamma = U_{i=1}^4\ \Gamma = U_z^i$ boundary, conjugate harmonic Functions (27) are connected with the help of $g_i(x, y) = 0,\ i = \overline{1,4}$ equations defining them.

Then, the problem to construct inverse conform mapping is identification of pair of conjugate harmonic functions of (27) type connected thorough Cauchy Riemann Equations:

$$\frac{\partial x}{\partial\xi} = \frac{\partial y}{\partial\eta}\cdot\frac{\partial x}{\partial\eta} = -\frac{\partial y}{\partial\xi}, (\xi, \eta) \in G_\xi, \tag{28}$$

complying with the equations of a boundary:

$$g_i\left(x(\xi, \eta), y(\xi, \eta)\right) = 0,\ i = \overline{1,4}, (\xi, \eta) \in \Gamma_\xi^i. \tag{29}$$

The boundary conditions should be added by equation of connection of boundary points with near-boundary ones using either:

– Cauchy Riemann Equations:

$$\frac{\partial x}{\partial\xi} = \frac{\partial y}{\partial\eta}, \frac{\partial x}{\partial\eta} = -\frac{\partial y}{\partial\xi}, (\xi, \eta) \in \partial G_\xi; \tag{30}$$

– equations of the grid area lines orthogonality to its boundaries which do not use equations of G_ξ boundary:

$$grad\ x \cdot grad\ y = 0, \text{ or } \frac{\partial x}{\partial\xi}\cdot\frac{\partial x}{\partial\eta} + \frac{\partial y}{\partial\xi}\cdot\frac{\partial y}{\partial\eta} = 0. \tag{31}$$

The algorithm of structural arrangement of difference grids with the help of the mathematical model is based upon approximation of Cauchy Riemann Equation set (20).

Calculate the conform difference grid relying upon the listed algorithms. Below, you can find a conform difference grid for the deposit resulting from the problem numerical solving (Fig. 5). Assume that G_z deposit area is a curvilinear rectangle (Fig. 5). The field boundaries may be prescribed by line equations (i.e. analytically) as well as by approximation through rather dense point table. In this case, the deposit area is a region with the four marked points *A*, *B*, *C*, and *D*, which will pass into vertices of the parametrical rectangle G_ξ during conform mapping.

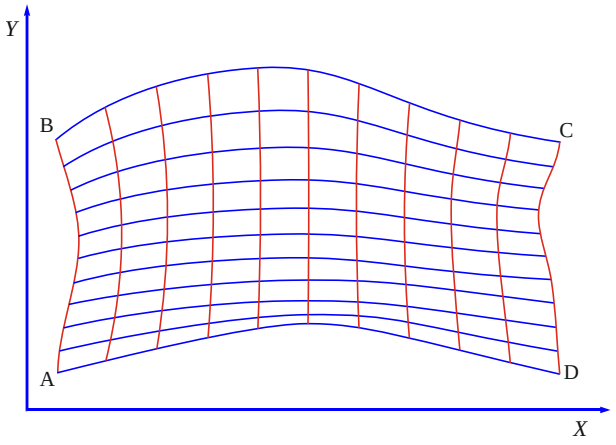

Figure 5. Deposit in the form of curvilinear rectangle

Construct conform difference grid for the deposit. The problem solving needs identification of a complex potential $\omega = \omega(z)$ or characteristic function $z = z(\omega)$ to define other characteristics. Under the conform mapping of G_ξ, and G_ω areas, coordinates of points are connected through Cauchy Riemann Equation $\frac{\partial x}{\partial \xi} = \frac{\partial y}{\partial \eta}, \frac{\partial x}{\partial \eta} = -\frac{\partial y}{\partial \xi}$ if boundary conditions are as follows:

$$g_i\left(x(\xi,\eta),\ y(\xi,\eta)\right) = 0, (x,y) \in \Gamma_z^i,\ i = 1,2,3,4\,. \tag{32}$$

In addition, the boundary conditions are added by orthogonality conditions for border points belonging to $\Gamma_z^1 \cup \Gamma_z^3 \cup \Gamma_z^4$:

$$\begin{aligned} &\frac{\partial x}{\partial \xi} = \frac{\partial y}{\partial \eta}, \frac{\partial x}{\partial \xi} = -\frac{\partial y}{\partial \eta}, (x,y) \in \Gamma_z^1 \cup \Gamma_z^3 \cup \Gamma_z^4; \\ &\frac{\partial x}{\partial \xi} \cdot \frac{\partial x}{\partial \eta} + \frac{\partial y}{\partial \xi} \cdot \frac{\partial y}{\partial \eta} = 0. \end{aligned} \tag{33}$$

Having implemented the numerical conform mapping of G_ξ rectangle onto G_z area, determine coordinates of $(\phi_{i0}, \varphi i0), i = \overline{0,n}$ for Γ_ω^2 and for the rectangle module.

Inverse problem is formulated to construct difference stream grid within the dimensionless variables of a complex potential $\omega = \phi + i\psi$, where ψ is the stream function.

Numerical solution of the problem is developed using floating of end nodes of the grid isolines along all boundaries $\Gamma_z^i, i = \overline{1.4}$ of G_z area. It is necessary to identify the complex potential $\omega = \phi(x, y) + i \cdot \psi(x, y)$, being analogous function within G_z area, which actual and imaginary parts are connected through Cauchy Riemann Equations:

$$\frac{\partial x}{\partial \xi} = \frac{\partial y}{\partial \eta}, \frac{\partial x}{\partial \eta} = -\frac{\partial y}{\partial \xi}, (x,y) \in G. \tag{34}$$

The inverse mapping problem is:

$$\frac{\partial x}{\partial \xi} = \frac{\partial y}{\partial \eta}, \frac{\partial x}{\partial \eta} = -\frac{\partial y}{\partial \xi}, (x,y) \in G_\omega; \tag{35}$$

$$\begin{aligned} &x(0,\psi) = 0,\ 0 \le y(0,\psi) \le H; \\ &y(1,\psi) = 0,\ l_1 \le x(1,\psi) \le l_2; \\ &y(\phi,0) = 0,\ 0 \le x(\phi,0) \le l. \end{aligned} \tag{36}$$

$$\begin{cases} y(\phi,0) = \phi - \phi, 0 \le x(\phi,0) \le l_2; \\ x_\phi \cdot x_\psi + y_\phi \cdot y_\psi = 0, (x,y) \in G_\omega. \end{cases} \tag{37}$$

It should be noted that the area module, considered before, may be provided with the determined content of a potential field. Assume that the area is a curvilinear rectangle. Conform mapping of the curvilinear rectangle G_z onto G_ξ rectangle should involve coincidence of their modules, i.e. $M_{G_z} = M_{G_\xi}$. Since $M_{G_\xi} = \Delta\phi / Q$, where Q is complete consumption of a matter, i.e. amber content; and $\Delta\phi$ is difference of potentials for *AD*, and *BC*, then:

$$Q = \Delta\phi / M_{G_\xi}\,. \tag{38}$$

In such a way, module is value being inverse to consumption. Having defined during numerical construction of the conform mapping the rectangle module as nh_1 / mh_2 ratio with some accuracy, will be able to identify Q consumption. Hence, the performed numerical conform mapping and the obtained difference grid have helped coordinate extraction chambers within the grid nodes.

Further, amber content within the deposit, limited by a curvilinear contour, is assessed through kriging method applied analogically earlier.

The research makes it possible to identify the best assessment of amber content within the analyzed block taking into consideration its percentage in samples taken inside contact zones one and two of the deposit. Moreover, the findings result in the improved accuracy of the reserve evaluation. They also open the way to develop models for assessment of mineral reserves within deposits inclusive of modern engineering as well as software solutions; among other things, it concerns neural networks, and AI capabilities to evaluate the reserves.

4. Conclusions

Complexes of hydromechanical testing help take samples corresponding to demands of accuracy and reliable visualization. The abovementioned defined the problem of amber placer reestimation towards increase and expansion of borders of minimal commercial and boundary value of amber content.

Various alternatives of the numerical conform mapping of parametrical rectangle have been considered; each of them involves difference grid within curvilinear rectangle, mapping the rectangle numerically in terms of the correspondent area boundary assignment either in square or in rectangular grid. Relying upon the results, a mathematical model has been developed to assess average content of a mineral both in the rectangular and curvilinear areas.

The analysis of the amber placer reserve evaluation, involved a discrete kriging method, makes it possible to assess average amber content in the rectangular zone and in the curvilinear one. The zones, being considered, tend to the

central well as for amber content and for wells arranged in contact zones one and two. The method includes search for the best assessment of amber content within the calculated block inclusive of its percentage in samples taken inside the block and out of it.

Author contributions

Conceptualization: ZM; Formal analysis: AK, RZ; Investigation: YM, AK, VK; Methodology: YM, VM, ZM; Project administration: ZM; Resources: VM, ZM; Validation: ZM, RZ; Writing – original draft: YM, VM, AK, ZM, VK; Writing – review & editing: YM, ZM, VK, RZ. All authors have read and agreed to the published version of the manuscript.

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Conflicts of interests

The authors declare no conflict of interest.

Data availability statement

The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

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Моделювання оцінки запасів корисних копалин методами дискретного крайгінгу

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Мета. Розробка ефективної модель оцінки запасів бурштинових розсипів в досліджуваному блоці з урахуванням його вмісту в пробах, розміщених як в середині, так і зовні блоку.

Методика. В роботі для отримання результатів булло застосовано комплекс наукових методів з системного аналізу, узагальнення, математичної статистики, математичного, ідеального та фізичного моделювання, натурного експерименту та прогнозування.

Результати. Обґрунтована методика оцінки запасів бурштинових розсипів із використанням можливостей дискретного крайгінга, який включає пошук найкращої оцінки вмісту бурштину в обраховуваному блоці з урахуванням його вмісту в пробах, розміщених як в середині, так і зовні оцінюваного блоку.

Наукова новизна. В результаті проведення досліджень встановлено, що аналіз результатів дискретного крайгінга дозволяє отримати оцінку середнього вмісту бурштину в прямокутній і криволінійній зонах, тяжіючих до центральної свердловини за вмістом бурштину, в центральній свердловині та у свердловинах першого і другого ореолів.

Практична значимість. Встановлення найбільш ефективного способу оцінки запасів бурштину підвищує достовірність прогнозування запасів бурштину з мінімальними затратами.

Ключові слова: *модель, бурштин, ефективність, оцінка, крайгінг, розсип, свердловина, корисна копалина*

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