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Application of the deterministic block theory to the slope stability design of an open-pit mine in Morocco

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Abstract

Purpose. Discontinuities in rock masses are natural fractures that delimit various block shapes and sizes, which can fall, slide or topple from the excavation and collapse under their own weight inducing probably severe damage. Thus, it is essential to carry out a block analysis before beginning any surface or underground excavation project. This paper proposes a methodology based on key block theory analysis to select the suitable slope of different discontinuous rock masses of an open-pit mine in Morocco.

Methods. At first, the main discontinuities of each bench are determined and projected onto a stereonet with a maximum dip angle of the excavation plane. Then, it is possible to identify the removable blocks by using the theorem of removability according to block theory. After that, a limit equilibrium analysis is performed to determine the failure mode and the friction angle required to stabilize the blocks. When the selected dip angle of the slope plane is found to be unsuitable, it is changed and reduced by one degree, and the same approach is repeated until the maximum safe slope dip angle is obtained.

Findings. The results of the proposed methodology based on key block theory analysis have shown that the maximum safe slope angles of the studied benches are in the range of 63-73°. When compared to the slope angles used in the mine, which are between 58-78°, the results of this study are close to in-situ conditions.

Originality. In this research, the maximum safe slope angle of fractured rock masses was optimized by eliminating slope angles inducing unstable blocks (key blocks) and by using the stereographic projection method of key block theory.

Practical implications. Using this methodology, stability of rock slopes in civil or mining-engineering projects can be designed or assessed when geotechnical data are very limited.

Keywords: *key block theory, slope stability, limit equilibrium analysis, discontinuous rock masses, stereographic projection*

1. Introduction

One of the most commonly used methods for extracting ore near the earth's surface is called open-pit mining (also known as open-cut, open-cast or strip mining). It contributes to production of large amounts of minerals, metals and other natural resources in the market. It is expected that production will continue to increase as the demand for natural resources only grows. An open-pit mine needs to be excavated at the steepest possible slope angle, not only to ensure the rock mass stability, but also to reduce the excavation cost. It is reported that a small change in the slope angle of 2-3° may correspond to a project cost of hundreds of millions of dollars [1]. Even with the right choice of the steepest slope angle, the probability of failure may be very high. That is why it is important to calculate the maximum safe slope angle and the associated probability of failure.

Since the rock mass contains many discontinuities splitting heterogeneous natural rocks into blocks of different sizes and shapes, the slope stability analysis of open pit mines needs to take into account the blocks that are apt to fall, slide, topple or rotate and, therefore, cause many damages to productivity. These blocks are so-called key blocks and the theory that deals with them is called block theory (known also as key block theory (KBT)). It was firstly introduced and developed by Goodman and Shi in 1985 [2] as a geometric set of analyses that determine where potentially dangerous blocks can exist in a geological material intersected by different orientations of discontinuities in three dimensions. It provides an effective tool to evaluate stability of different types of jointed rock masses, especially hard ones [3]. According to this theory, the blocks are assumed to be rigid, and the discontinuities are perfectly planar. By applying the theorem of finiteness and removability, it is possible to evaluate the stability of a surface or underground excavation. Warburton used a vector method for the stability analysis of an arbitrary polyhedral rock block with any number of free surfaces [4]. Delport and Martin demonstrated a completely new key block characterization based on mathematical analysis and presented an identification algorithm. The characterization based on an alternative Tucker's theorem and algorithm simply requires performing a few pivot operations and checking the sign of a linear programming variety [5]. Heliot developed a new method to generate 3D block structure surrounding rock excavations using a programming language called Block Generation Language (BGL). The tectonic history of the rock mass was also incorporated in the pro-

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gram [6]. Lin and Fairhurst proposed a procedure for 3D analysis of the static stability of an assembly of polyhedral rock blocks around an underground excavation [7].

To deal with rotational or toppling failure of rock blocks, Mauldon and Goodman developed a vector technique for analyzing the possibility of rotational failure of key blocks [8]. Then, Tonon generalized Mauldon's and Goodman's vector analysis of key block rotations to take into consideration loads of any kind (such as reinforcement, seepage, and external water pressure) by applying rigid body dynamics [9]. Recently, Azarafza et al. proposed a semi-distinct element algorithm, based on KBT, for stability analysis of the three basic toppling failures: block toppling, flexural toppling and block-flexural toppling. By using analytical description of the modified SORM-FOE for the estimation of safety factor values for discontinuous rock slopes, the algorithm was constructed and coded using the python software and employed to high-order calculation trending loops [10].

Lu (2002) presented an algorithm based on topology for identification of 3D rock block formed by discontinuities. The algorithm called Rock Block Generator can recognize the rock block structure and provide useful input for different methods such as discrete element method, KBT, and Discrete deformation analysis [11]. González-Palacio et al. (2005) discussed the problem of geometric identification of non-pyramidal key blocks and also developed ubiquitous method that allows identifying tetrahedral and pentahedral key blocks using geometric dislocation in relation to the direction of tunnel axis and the method was implemented in ASTUR program in order to simplify the analysis [12]. Menéndez-Díaz et al. (2009), in another article used the concept of non-pyramidal key blocks in optimizing an underground marble mine in order to reduce the number and volume of key blocks formed by three and four discontinuity planes and thus increase the safety factor [13].

Elmouttie, Poropat, and Krähenbühl (2010) have made additional attempts to create a modeler that could manage several curved, finite persistent discontinuities [14]. The developed modeler was used to analyze the stability of underground excavations and proved to be more advantageous than the approach proposed by Menéndez-Díaz et al. (2009). To consider force interactions of the neighboring batches of key blocks, Fu and Ma (2014) enhanced the KBT and suggested a force transfer algorithm. To optimize the design of the rock support system, a two-step safety check was also used [15]. Based on the results, the proposed rock support design method was found more rational and realistic. In the paper of Li et al. (2012), the features of non-convex blocks with complex combinations of free planes are thoroughly examined using the traditional block theory [16]. Firstly, a non-convex block is viewed as a collection of convex blocks, then the requirements for the finiteness and removability of non-convex blocks are put forth, and finally, the identification technique is developed and some examples are used to validate it.

KBT has undergone many extensions and one of the main extensions is called the Key Group Method (KGM). Proposed by Yarahmadi Bafghi and Verdel (2003), KGM considers not only a single isolated key block, but also an assembly of collapsible blocks when analyzing slope stability. The results of this method were compared to those obtained by distinct element method and proved to be more realistic than the classical KBT [17]. Furthermore, Noroozi, Jalali, and Yarahmadi-Bafghi (2012) extended KGM to 3D key-group for slope stability analysis because of the conservational results of 2D analysis [18]. To examine general movable blocks with numerous structural planes that cannot be analyzed using classical KBT, an optimization model for resolving the safety factor of blocks was proposed and tested in the study of Sun, Zheng and Huang (2015). It was assumed in the model that both the safety factor and the normal stress on the slip surface are independent variables [19]. Zheng, Xia, and Yu (2015) have developed a unified method to analyze the stability and removability of rock blocks, given the cracking of rock bridges between the blocks, and non-removable blocks are not considered stable as in KBT. The results of this study show that non-removable blocks are not always stable and in some cases, present a greater risk than removable ones. The unified analysis approach can be considered as a better iteration of the vector method [20]. Zhang et al. (2020) introduced the basic analysis process of the General Block (GB) method and applied the method to evaluate the stability of rock blocks in an underground hydropower station under the Three Gorges Project [21]. In a recent paper, Zhu, Azarafza and Akgün (2022) provided a framework for classifying rock blocks in accordance with the concepts of KBT. In this paper, high-resolution photos of 130 slope masses were analyzed using a deep Convolutional Neural Network (CNN) method. According to Goodman's idea, three different types of rock blocks – key blocks, trapped blocks, and stable blocks – have been classified using a recognition method. The loss function, the root mean square error (RMSE) and the mean square error (MSE) were used to validate the proposed prediction model [22].

In order to obtain more comprehensive information about the excavation stability and the key block failure probability, many researchers have conducted probabilistic analysis to deal with variability and uncertainty of rock mass properties [23]-[26]. The difficulty of these analysis remains in estimating the failure probability of rock blocks due to complexity of mathematics involved. That is why many researchers use different techniques, such as Block Failure Probability, FOSM, PBTAC, etc., to overcome this difficulty [27]-[36].

From all of the above studies, it appears that KBT has undergone many improvements to deal with the heterogeneity and complexity of the rock masses. However, it should be noted that KBT is used in the stability analysis in many case studies around the world due to its simplicity, efficiency and proven validity as the results are more realistic [37]-[43].

In the present paper, traditional key block theory is applied to select the maximum safe slope angle of benches of an open pit mine in Morocco. Firstly, the discontinuities data collected by manual mapping was carefully analyzed to obtain the main joint sets that penetrate the geological rock material at the subjected benches. Secondly, KBT, implemented in KBslope software of Pantechnica workshop, was used to perform stereographic projection of the main discontinuities and MSSA to identify the removable blocks according to Goodman and Shi approach. Finally, a limit equilibrium analysis is performed to determine the failure mode of the removable block and to decide on the selection of MSSA.

2. Methods

2.1. Block theory

2.1.1. Assumptions

Before conducting KBT analysis, some basic assumptions should be satisfied:

- all the discontinuities are assumed to be perfectly planar;
- surface discontinuity is considered as an infinite plane;

– each block existing in the rock mass is assumed to be rigid;

– the slope plane (or excavation plane) and discontinuities are defined as input parameters in the study (variation of joint set orientation is not considered in the deterministic KBT).

2.1.2. Types of blocks

According to block theory, rock blocks can be classified into five types as shown in Figure 1a. Infinite blocks are not excavation hazard due to their inability to internally crack. Finite blocks are determined by joint and free planes and are divided into removable and non-removable blocks. Non-removable block has tapered shape and therefore cannot move out of the rock mass. While removable ones are non-tapered and can be divided into stable blocks, stable with sufficient friction and key blocks.

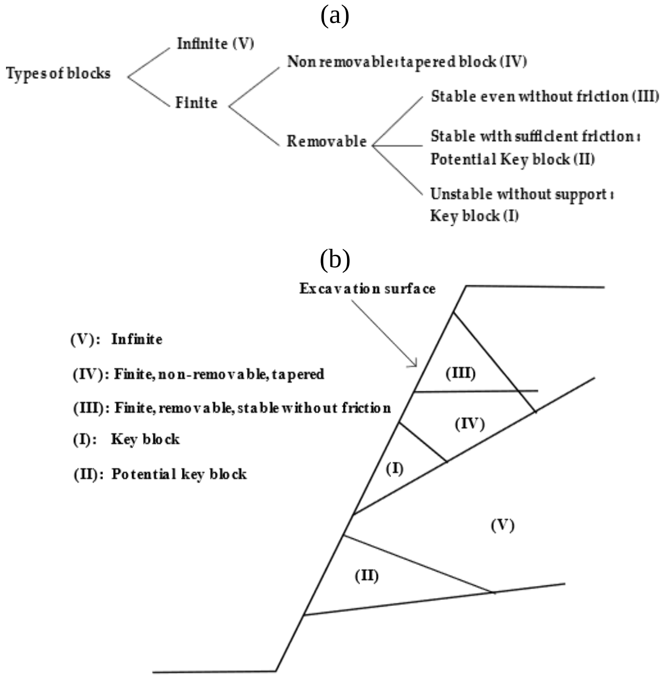

Figure 1. Block classification: (a) chart; (b) in a surface excavation

Figure 1b shows five types of surface excavation blocks in two dimensions. The key block (type I) is the most critical rock block due to its ability to slide or fall.

2.1.3. Finite and removable blocks

According to block theory, the intersection of discontinuity and rock slope half spaces defines the blocks. The intersection of discontinuity half spaces delimits the joint pyramid (*JP*). The slope plane divides the space into two half spaces: excavation pyramid (*EP*) and space pyramid (*SP*).

The intersection of joint pyramid and excavation pyramid determines the block pyramid (*BP*):

$$BP = JP \cap EP . \tag{1}$$

According to the finiteness theorem, the block is considered finite only if the BP is empty, i.e:

$$BP = \varnothing \text{ or } JP \subset SP . \tag{2}$$

This also means that the block is infinite if the *BP* is not empty, or:

$$JP \not\subset SP . \tag{3}$$

Even though the block is finite, it can be non-removable (in case of tapered block). Therefore, the removability theorem should be applied: a block is removable if:

$$BP = \varnothing \text{ and } JP \neq \varnothing , \tag{4}$$

and becomes non-removable if:

$$BP = \varnothing \text{ and } JP = \varnothing . \tag{5}$$

2.1.4. Failure modes

Once removable blocks have been identified, their stability should be analyzed to determine the mode of their failure, whether by lifting (falling), sliding along a single discontinuity plane or sliding along the intersection of two planes.

Lifting occurs when the sliding direction follows the resultant force and there is no contact with discontinuity planes. Sliding along a single plane takes place when the sliding direction lies on within a plane. Sliding along the intersection of two planes occurs when the sliding directions are contained simultaneously in two discontinuity surfaces.

To determine the removable block resultant force direction, a mode analysis developed in Chapter 9 of block theory [2] will be used. This will identify stable blocks (Type III) from potential key blocks (Type II and I).

2.1.5. Sliding equilibrium stability analysis

This kind of analysis was firstly introduced and performed by many researchers for tetrahedral shapes of blocks [44]-[49]. It aims not only to determine the sliding mode of the removable block, but also to identify real key blocks (Type I) from potential key blocks (Type II), when the friction angle of each joint is already known.

Sliding equilibrium analysis can be carried out using vector method or graphically by projecting possible sliding and falling regions onto a stereonet that contains a removable block delimited by discontinuity half spaces. Inside each obtained failure region, symbols are used to indicate the mode of failure (*Ji* for sliding along a plane *Ji*, *Ji*/*Jj* for double plane sliding along the intersection of *Ji* and *Jj*, 0 for falling). Friction contours are constructed to estimate the friction angle needed to achieve the stability of the removable block entering the sliding mode.

2.2. Methodology for the case study

Figure 2 displays the flowchart of the detailed procedure for the selection of MSSA based on the original KBT using the stereographic method. It should be noted that in order to reduce the number of key blocks and achieve stability, it is necessary to reduce the slope angle of the excavation plane or add support to the key blocks.

In this paper, the slope angle has been reduced to achieve the stability condition, since artificial supports are not frequently used in the mine.

2.3. Case study

2.3.1. Geographical location of Tazalaght mine

Tazalaght is one of the largest open pit mines in Morocco where copper ore is mined. It is located in the Western Anti-Atlas of Morocco, 150 km SE from Agadir and about 30 km from Tafraout. The mine is bounded by north latitude 29°44′42″ and 29°45′4″ and east longitude -8°43′37″ and -8°43′5″.

The mine is managed by Akka Gold Mining (AGM), an affiliate of Managem Group. It was originally operated as a gold mine, but since 2007, the Akka site has been progressively converted to copper, thanks to the discovery of new copper resources in the Tazalakht and Ouansimi deposits, which came into production in 2017.

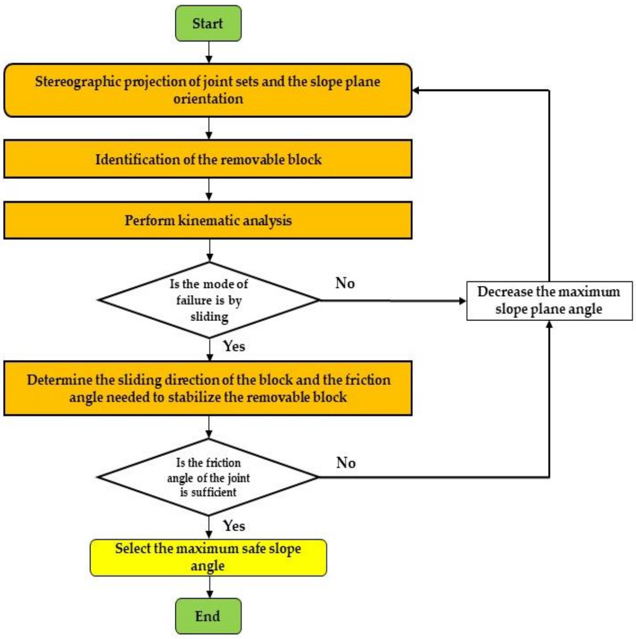

Figure 2. Flowchart for the selection procedure of MSSA based on KBT

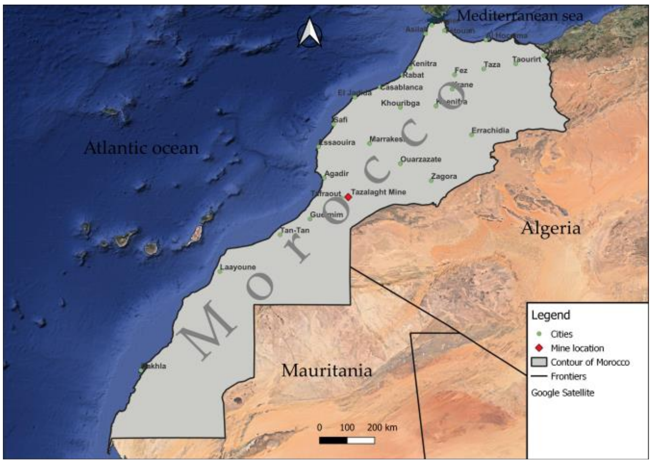

Figure 3. Geographical location of Tazalaght open pit mine

Figures 3 and 4 show, respectively, the mine geographical location and the slopes that require stability analysis in order to exploit the mineralization, given the safety standards and economic considerations, as the project deepens in the future.

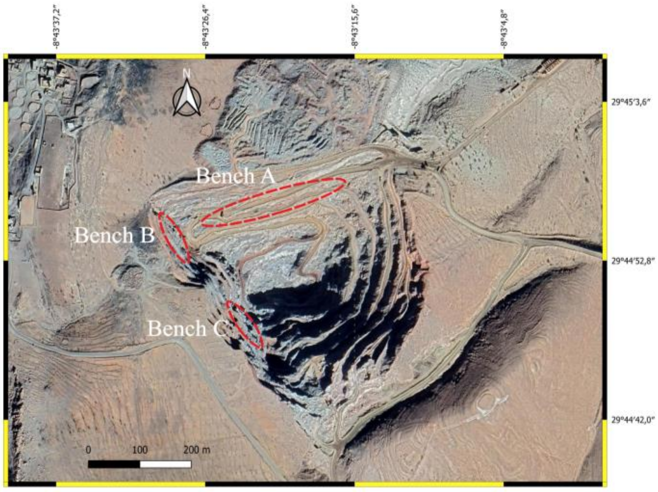

Figure 4. Geographical location of the studied benches in the mine

2.3.2. Geological and geotechnical setting

The Tazalaght deposit, located on the eastern border of Ait Abdellah inlier, occupies an anticline structure oriented northeast (NE) with a length of approximately 500 m. As shown in Figure 5a, b, it is composed of two major lithological formations: Neoproterozoic formations and early Cambrian unit (also called Tata-Taroudant Group). The Neoproterozoic formations consist of quartzites unconformably overlain by conglomerates that may represent a succession of paleoshoals and paleobasins [50], [51]. While the early Cambrian unit consists of Base Series (composed of alternating sandstones and siltsones) and Cambrian carbonates of the "Lower Limestone Series," including the thick "Tamjout dolomite" layer [52]-[54].

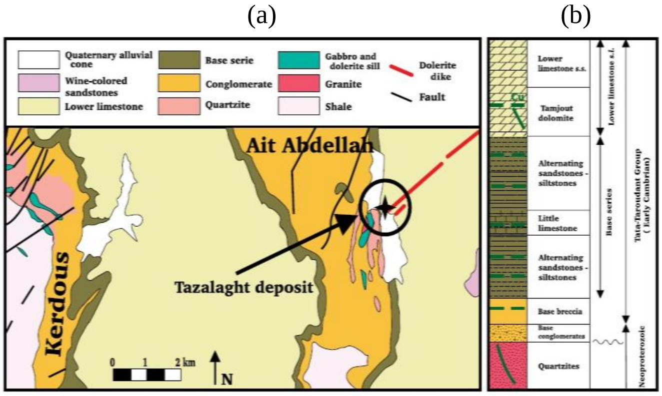

Figure 5. Geology of the study area: (a) geological map of Ait Abdellah, Kerdous inliers and Tazalaght location [51]; (b) stratigraphic column of Tazalaght deposit, indicating the richest copper layers [51]

In line with the International Society for Rock Mechanics (ISRM) standards, intact dolomite and base series are hard with uniaxial compressive strength (UCS) between 111.3 and 209.7 MPa, respectively. Quartzites and the Breccia base are considered medium with UCS ranging from 50 to 79.4 MPa. Therefore, each block of the studied rock masses can be considered rigid according to KBT.

2.3.3. Orientation of the main discontinuities

Two hundred and thirty-six discontinuities have been manually mapped at four benches of the mine (A1, A2, B and C). Tables 1, 2, 3 and 4 provide the dip directions of slope planes and some geometric and mechanical characteristics of major and minor discontinuities that transverse the rock mass, respectively, at benches A1, A2, B and C.

According to the methodology presented in section 2.2, the orientation of joint sets and slope planes of each bench will be considered as inputs to the block theory analysis to determine the suitable MSSA.

Table 1. Joints and slope orientation of bench A1

Plane	Dip angle	Dip direction	Discontinuity condition	Friction angle (°)
J1	70	88	Smooth and planar	10-20
J2	70	163	Smooth and planar	10-20
J3	58	253	Smooth and planar	10-20
J4	76	345	Smooth and planar	10-20
Slope plane	–	153	–	–

Table 2. Joints and slope orientation of bench A2

Plane	Dip angle	Dip direction	Discontinuity condition	Friction angle (°)
J1	84	179	Smooth and planar	10-20
J2	86	84	Smooth and planar	10-20
J3	82	236	Smooth and planar	10-20
J4	12	162	Smooth and planar	10-20
Slope plane	–	153	–	–

Table 3. Joints and slope orientation of bench B

Plane	Dip angle	Dip direction	Discontinuity condition	Friction angle (°)
J1	85	271	Smooth and planar	10-20
J2	85	222	Smooth and planar	10-20
J3	35	196	Smooth and planar	10-20
Slope plane	–	49	–	–

Table 4. Joints and slope orientation of bench C

Plane	Dip angle	Dip direction	Discontinuity condition	Friction angle (°)
J1	82	331	Rough and irregular	10-20
J2	66	77	Rough and irregular	10-20
J3	75	24	Rough and irregular	10-20
Slope plane	–	20	–	–

In the first step of key block analysis, the major joint sets (J1, J2, J3) affecting the rock masses at the bench face, will be represented on the streonet with slope plane orientation to identify the removable block. Then, this block will be analyzed by the limit equilibrium method to verify its stability according to block theory.

3. Results and discussion

3.1. Bench A1 and A2

For bench A1, the stereographic projection of the major discontinuities J1, J2, J3 and the slope plane for a dip angle 90º on a lower focus projection is shown in Figure 6a. The only removable block is JP 101, because it is entirely outside of the slope plane circle. To determine the mode of this block failure, the sliding equilibrium regions presented in Figure 6b were constructed according to the method discussed in the subsection of sliding equilibrium region. Figure 6b shows that JP 101 will slide along J2, and the friction required to ensure stability of this block is about 70º. In fact, it seems that this block is a real key block and will really slide along J2 because the real friction of the sliding plane is not sufficient to avoid movement of this block. Therefore, the slope dip angle should be reduced until MSSA is achieved.

Table 5 displays the results of dip angle effect on the bench A1 stability and demonstrates clearly that MSSA of bench A1 is about 67º.

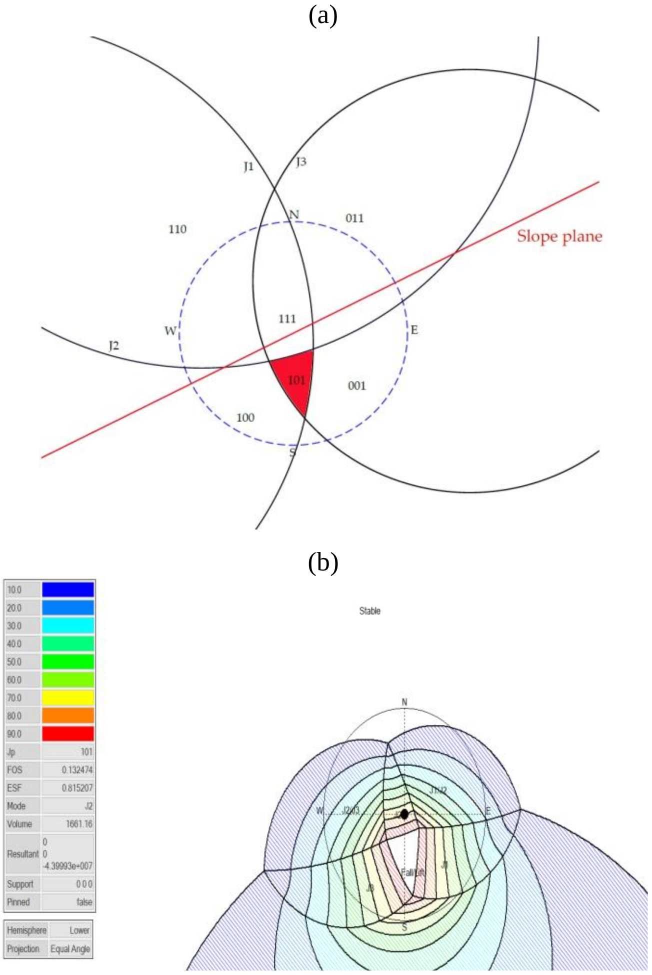

Figure 6. Results of block theory analysis at bench A1 for a slope angle 90°: (a) stereographic projection of the main discontinuities and slope plane for a slope angle 90° (on a lower hemisphere projection); (b) sliding equilibrium regions and friction contours for JP101 (on a lower hemisphere projection)

Table 5. Effect of slope dip angle variation on slope stability of bench A1

Dip angle (°)	Removable JP	Type of the block	Sliding plane (s)	Friction needed to stabilize
90	101	Key block	J2	70
85	101	Key block	J2	70
80	101	Key block	J2	70
75	001	Key block	J1/J2	72
70	001	Key block	J1/J2	72
67	000	Stable with sufficient friction	J1/J3	9
66	000	Stable with sufficient friction	J1/J3	9
65	000	Stable with sufficient friction	J1/J3	9
60	000	Stable with sufficient friction	J1/J3	9
55	000	Stable with sufficient friction	J1/J3	9

Figures 7a, b illustrate, respectively, the stereographic projection (J1, J2 and J3 and slope plane for a dip angle 67º) and sliding equilibrium regions with friction contours of JP 000. Hence, JP 000 will remain stable as the real friction along joints J2 and J3 is above 9º and sufficient to avoid block instability.

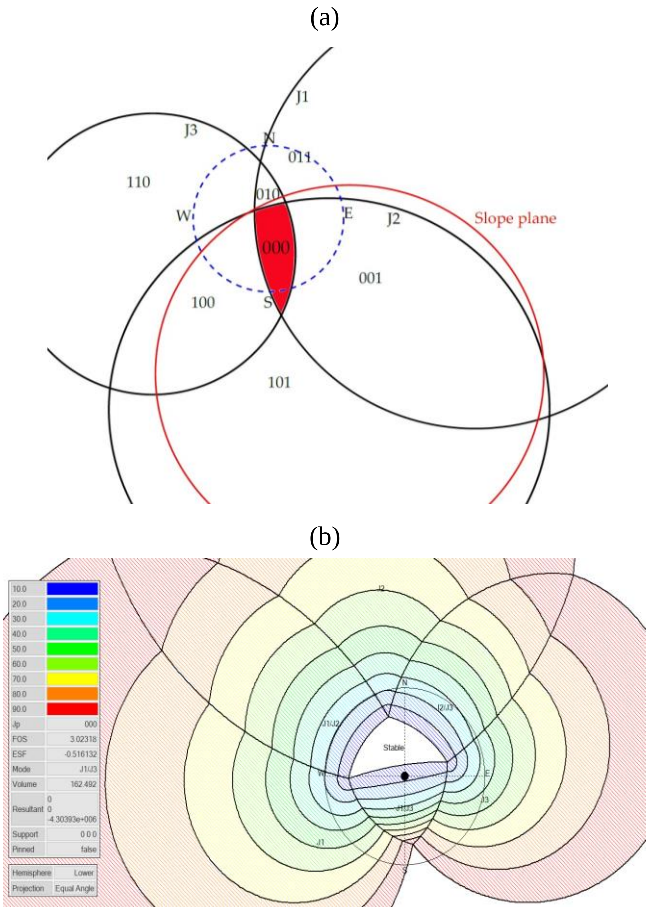

Figure 7. Results of block theory analysis at bench A1 for MSSA = 67°: (a) stereographic projection of the main discontinuities and slope plane for MSSA = 67° (upper hemisphere projection); (b) sliding equilibrium regions and friction contours for JP000 (lower hemisphere projection)

For bench A2, the first stereographic projection of the major discontinuities J1, J2, J3 and the slope plane for a dip angle 90º, summarized in Table 6, revealed that the only removable block is JP 011. Furthermore, the mode of this block failure is sliding along J1 and the friction required to ensure stability of this block is about 84º. Therefore, the dip angle of bench A2 decreased until MSSA was achieved, because the real friction angle along J1 is not sufficient to ensure stability by friction. It was found that MSSA is 66º, because at this slope angle JP 100 is a stable block even without friction (type III according to KBT) and there is no need for artificial support.

Table 6. Effect of slope dip angle variation on slope stability of bench A2

Dip angle (°)	Removable JP	Type of the block	Sliding plane (s)	Friction needed to stabilize
90	011	Key block	J1	84
85	001	Key block	J1/J2	84
80	000	Key block	J2/J3	48
75	000	Key block	J2/J3	48
70	000	Key block	J2/J3	48
67	000	Key block	J2/J3	48
66	100	Stable	–	–
65	100	Stable	–	–
60	100	Stable	–	–
55	100	Stable	–	–

3.2. Bench B and C

The results of key block theory analysis for different slope angles at bench B are summarized in Table 7. It can be seen that for a dip angle of about 90º, the stereographic projection of the major discontinuities J1, J2, J3 and the slope plane orientation show that the only removable block is JP 110. In addition, the mode of this block failure is sliding along the intersection of the joint sets J2 and J3. It was also revealed that the friction required to ensure the stability of this block must be above 12º. Therefore, MSSA can be achieved at a dip angle of 90º, because the real friction angle along J1 is sufficient to ensure the block stability. However, by selecting a slope angle of about 73º, the rock mass of this bench will remain stable, since JP 010 is a removable, but a stable block (type III) and there will no need for friction of the joints. Hence, MSSA will be selected at a slope angle of 73º in order to prevent any mode of failure.

Table 7. Effect of slope dip angle variation on slope stability of bench B

Dip angle (°)	Removable JP	Type of the block	Sliding plane (s)	Friction needed to stabilize
90	110	Stable with sufficient friction	J2/J3	12
85	110	Stable with sufficient friction	J2/J3	12
80	110	Stable with sufficient friction	J2/J3	12
75	110	Stable with sufficient friction	J2/J3	12
74	110	Stable with sufficient friction	J2/J3	12
73	010	Stable	–	–
70	010	Stable	–	–
65	010	Stable	–	–
60	010	Stable	–	–
55	010	Stable	–	–

For bench C, the results of key block theory analysis for different possible dip angles are presented in Table 8. It is indicated that for a dip angle of about 90º, the joint pyramid coded 110 is the only removable block. In addition, this block presents a risk of wedge sliding along the intersection of the joint sets J1 and J3 if the friction angle of the sliding planes does not exceed 89º or if artificial support is not added. Therefore, the slope face angle was modified several times until MSSA was obtained. It was found that MSSA is about 63º, because at this slope angle the stereographic projection of the joint sets J1, J2, J3 and the excavation plane revealed that JP 000 is a finite and removable joint pyramid and is considered as a stable block (type III) according to the limit equilibrium analysis.

The results of block theory analysis for the selected benches A1, A2, B and C indicate that the maximum safe steepest angles are 67, 66, 73, 63º respectively. While these bench faces were excavated in the mine at 68, 68, 78, 58º respectively. Therefore, it can be concluded that the results of this study are close to reality in the field. However, these results were performed under the assumption that the rock masses are loaded only by the gravitational force. Hence, further analysis of limit equilibrium should be performed by integrating other forces such as water forces, seismic forces, etc.

Table 8. Effect of slope dip angle variation on slope stability of bench C

Dip angle (°)	Removable JP	Type of the block	Sliding plane (s)	Friction needed to stabilize
90	110	Key block	J1/J3	89
85	110	Key block	J1/J3	89
80	110	Key block	J1/J3	89
75	000	Key block	J1/J2	76
70	000	Key block	J1/J2	76
65	000	Key block	J1/J2	76
63	000	Stable	–	–
60	000	Stable	–	–
55	000	Stable	–	–

In addition, these results should be compared to other methods of slope stability analysis to verify other failure modes, such as toppling failure.

4. Conclusions

This paper proposes a methodology for the selection of MSSA for different jointed rock masses of an open pit mine in Morocco based on a deterministic key block theory. First, a summary of KBT has been presented. Then, the statistical analysis of discontinuities affecting each bench of the mine revealed the existence of three joint sets that delimit rock blocks. The stereographic projection of the joint sets and slope orientation for a maximum slope angle has shown removable joint pyramids that require kinematic analysis in order to determine the failure mode and identify the block type. When sliding is a potential mode of block movement, the friction angle of the sliding plane can be evaluated by constructing friction contours according to KBT, and the optimal friction needed to stabilize the block can also be determined.

This research indicates that reducing the slope angle of each studied bench considerably reduces the formation and failure of key blocks, and thus the stability of the bench is achieved. Moreover, when the failure mode is sliding along a single or a double plane, the friction angle of the joint responsible for sliding is a key parameter in the stability analysis of the rock block, and in some cases, when is it satisfactory, can provide MSSA. As results, MSSA at the four studied benches A1, A2, B and C is, respectively, 67, 66, 73, 63º, which provides the stable rock blocks with sufficient friction.

However, in some cases, even the slope angle decreases at the lowest value, key blocks may be formed, causing severe damage. Therefore, it is important to add artificial support in order to increase the slope angle of the excavation and achieve stability. Furthermore, the current research has been conducted based on some deterministic parameters of joint orientation (dip and dip direction) and slope plane direction angle of the benches. In fact, the spatial distribution and uncertainty of intact rock properties and discontinuities can be evaluated from the stability of key blocks. In this context, probabilistic analysis approaches may provide a better solution for this issue.

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Застосування детерміністичної теорії блоків до розрахунку стійкості укосів відкритого кар'єру в Марокко

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Мета. Розробка нової методології, що базується на аналізі теорії ключових блоків для вибору відповідного ухилу різних переривчастих масивів гірських порід на відкритому кар'єрі в Марокко й попередження їх руйнувань.

Методика. Спочатку визначаються основні порушення суцільності кожного уступу та проєктуються на стереосітку з максимальним кутом падіння площини виїмки. Потім можна ідентифікувати блоки, що видаляються, використовуючи теорему про усунення відповідно до теорії блоків. Після цього виконується аналіз граничної рівноваги для визначення режиму руйнування та кута тертя, необхідного для стабілізації блоків. Коли вибраний кут нахилу площини укосу виявляється невідповідним, його змінюють і зменшують на один градус, і той же підхід повторюється доти, доки не буде отримано максимально безпечний кут нахилу укосу.

Результати. Результати використання запропонованої методики, що базується на аналізі теорії ключових блоків, показали, що максимальні безпечні кути нахилу укосів, що вивчаються, знаходяться в діапазоні 63-73°. Якщо порівняти з кутами укосу, що використовуються в шахті, які становлять від 58 до 78°, то результати цього дослідження близькі до натурних умов.

Наукова новизна. Максимальний безпечний кут нахилу масивів тріщинуватих гірських порід оптимізовано шляхом усунення кутів нахилу, що спричиняють появу нестійких блоків (ключових блоків), і з використанням методу стереографічного проєктування теорії ключових блоків.

Практична значимість. Використовуючи запропоновану методологію, можна спроєктувати чи оцінити стійкість укосів гірських порід у цивільних чи гірничо-технічних проєктах, коли геотехнічні дані є дуже обмеженими.

Ключові слова: *теорія ключових блоків, стійкість укосів, аналіз граничної рівноваги, переривчасті масиви гірських порід, стереографічне проєктування*