Predictive geometrization of grade indices of an iron-ore deposit

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Abstract

Purpose is development of the methods to predict indices of iron-ore deposits relying upon the improvement of available techniques as well as formulation of new geometrization procedures and identification of the most adequate decision-making way to assess geological data as the basis for geometrization and prediction.

Methods are to develop a self-organizing prediction algorithm based upon combination of the available techniques and formulation of new mathematical methods; consider various means to assess them in the context of iron-ore deposit; and select the most efficient one. Use of geostatistical methods makes it possible to evaluate and process output geological information. The methods help assess mineral reserves of a mining enterprise.

Findings. Dependencies of magnetite ore content upon geological factors have been derived in the context of an open pit of PIVDGZK JSC. The deposit has been geometrized; predictive mining and geometric model of the deposit site has been developed. Factors have been determined influencing the distribution nature of the indices. Graphs to arrange grade indices of the deposit have been constructed. The graphs have helped predict their placement within the deposit.

Originality. A method to predict mining and geological indices of iron-ore deposit has been developed relying upon a self-organizing algorithm. Correlation between grade indices of minerals and different geological factors has been determined making it possible to describe spatial distribution of grade indices of the deposit.

Practical implications. Geometrization methods for iron-ore deposits have been formulated. The methods help schedule mining operations accurately while improving their efficiency. The developed predictive self-organizing algorithm is the flexible tool used for various mining and geological conditions to provide scheduling and assessing of different mining methods. The self-organizing as well as geostatic evaluation techniques is quite a promising research tendency.

Keywords: self-organizing algorithm, geostatic methods, reserves, geometrization, grade indices, prediction.

1. Introduction

Scheduling of operations as well as achieving of output with the specified content of useful component is one of the most important missions of mining. The abovementioned should involve systematic designing and modeling of a mining enterprise operations and production processes at the opening stage [1-4]. To solve the problem, the fullest idea of mining and geological conditions of a deposit is required in addition to its geometry and spatial arrangement of excavating and technological indices. The data obtaining is supported by the methods of subsoil geometrization [5-8].

Various types of deposit geometrization are intended to solve mining problems both graphically and analytically. Determination of rock mass stability and its stress-strain state is quite an important problem solved with the help of geometrization methods [9-13]. Most of all, geometrization of geological forms and spatial distribution of rock mass characteristics relies upon information concerning subsoil arrangement of mining and geological indices. Use of extraction and geometric methods to develop models of rock mass characteristics and indices helps recommend certain measures aimed at the rock maintenance as well as stable mineral extraction under different mining and technological conditions [14-16]. Indices with similar distribution nature define both the procedure and the efficiency of drilling and blasting operations [17], [18], as well as transportation issues [19], [20]. Geometrization is also vital for complex monitoring of rock mass state in the context of large-scale mining [21], [22]; in turn, the abovementioned identifies the methods controlling different mining processes [23], [24].

Geometrization of mineral deposit is based upon the data on geological, geochemical, geomechanical, and other fields characterizing various features and indices of rock mass, and sources of georesources. Reliability and accuracy of the information, its processing objectivity, and mining and geological indices as well as mineral occurrence conditions, determined as a result, will influence mining efficiency and the potential for selective extraction of the deposit mine-
Iron ore deposits demonstrate high discontinuity level of arrangement of its indices described on the basis of multidimensional random geochemical field resulting in complex determination of their geometric location within a deposit [33], [34]. To some extent, geometrization process of an iron ore deposits is always connected with a predictive spatial position of its indices. For the purpose, various analytical and graphical techniques are applied differing in their accuracy and performance; the abovementioned influences directly on the efficiency of the mining scheduling.

Analysis of the current methods of mining and geometric assessment of grade characteristics of mineral deposits helps conclude that geostatic kriging methods are the most acceptable ones for iron ore fields [35]-[39] since they make it possible to evaluate output geological data, accuracy of their formation, and the data applicability for geometrization and prediction of the deposit indicators. The techniques are applied to obtain analytical and graphical mining and geometric deposit models making it possible to improve scheduling efficiency of mining operations while advancing their production indicators.

The known predictive algorithms are based upon mathematical statistics or upon a ready-made dependence being adapted to the developed model by means of the required minimum changes which cannot produce the desired results while considering rather complex processes inclusive of the geological ones. In part, self-organizing predictive algorithms do not have any disadvantages; however, they are somewhat imperfect [40]-[42]. Such methods make it possible to take into consideration the factors influencing heavily accuracy of deposit evaluation as well as prediction of its grade indices. Generally, the processes take place owing to the fact that during the work calculation algorithm may optimize itself, i.e. become self-organized. Hence, self-organized methods are more advantageous than classical geometrization and prediction methods. However, heuristic approaches also have a part of the listed disadvantages, demand a great deal of work to group the data, and have a small reliability range.

Consequently, proceeding from the analysis of scientific sources, one can conclude that mining and geometric prediction of grade mineral indices is the especially critical aspect concerning geometrization of iron ore deposits to solve both future and current scheduling problems for organization of the most efficient activities by mining enterprise in a mode of ore grade averaging while improving rationalization of a deposit development. Studies, aimed at formulation of mining and geometric method predicting grade indices of iron ore deposits, are rather topical. As the analysis explains, heuristic predictive methods and geostatic approaches assessing grade indices of iron ore indices are the most promising ones. The paper concerns formulation of a mining and geometric method to predict grade indices of iron ore deposits based upon the improved heuristic and geostatistical approaches.

2. Methods

The authors have developed multidimensional heuristic predictive algorithm based upon arbitrariness of arbitrary-degree polynomial and determination of the optimal (1)-(3) function type [43]:

\[
f_i = \left[ c_i \left( a_{11}^i x_1^p + b_{11}^i x_1^q \right)^p \left( a_{12}^i x_2^p + b_{12}^i x_2^q \right)^p \right] \times \ldots \times \left( a_{1n}^i x_n^p + b_{1n}^i x_n^q \right)^p \left( a_{21}^i x_1^p + b_{21}^i x_1^q \right)^p \times \ldots \times \left( a_{2n}^i x_n^p + b_{2n}^i x_n^q \right)^p \right];
\]

\[
F_i (x_1, x_2, \ldots, x_n) = d_i \left[ f_1 (x_1, x_2, \ldots, x_n) + \ldots + f_n (x_1, x_2, \ldots, x_n) \right] + e^p,
\]

\[
P (x_1, x_2, \ldots, x_n) = g^p \left[ F_1 (x_1, x_2, \ldots, x_n) + \ldots + F_n (x_1, x_2, \ldots, x_n) \right] + h^p.
\]

In the context of Expressions 1, 2 and 3, \( x_1, x_2, \ldots, x_n \) values are the arguments upon which the predicted parameter \( P \) may depend. All the known parameters, being within the basic data set where the predictive function is under construction, are assumed as arguments. In this case, they are geological indices in a point with the known \( P \) parameter as well as the planned and altitude coordinate sampling point. In addition, the arguments can be only the values known within the simulated data set for which \( P \) parameters should be predicted. \( a_i, b_i, c_i, d_i, e_i, g, \) and \( h \) values are numerical coefficients. \( p \) exponents are functions being similar in their structure to \( P (x_1, x_2, \ldots, x_n) \) function. Despite equivalent terms, \( p \) values in Expressions 1-3 may have different values. \( n \) value is the number of arguments. \( m \) value, being a part of Expression 3, will be explained while considering a procedure of the predictive function construction.

The basic data set has been used to identify values of the predicted index \( P_m \). Specify the values, calculated with the help of the predictive function, as \( P_i \). Their difference is:

\[
\Delta M_i = P_m - P_{i_m};
\]

The predictive function is considered as the constructed if

\[
\sum |\Delta M_i| \text{ within the basic set of output data is minimal in terms of each unit of the output data.}
\]

\[
\sum |\Delta M_i| = \text{min}.
\]

The unit of output data is understood as a set of arguments within a point with the known \( P_m \) value.

In the simplest case, expression (1) may involve only one Expression of type 2. In this context, the Expression of type 2 may involve only one Expression of 3 type which, in turn, involves only one factor in round brackets and \( c_i \) coefficient. Hence, start identifying a predictive function from the coefficient. It seems to be irrational idea at the very beginning of the algorithm; nevertheless, it becomes very expedient while identifying function type of degree \( p \) indicators since the detailed examination of Expressions 1, 2 and 3
shows that order of \( p \) degrees may grow endlessly. The number of factors and additions in the predictive function may also be endless.

Initial \( c_1 = 1 \) value is set to \( c_i \) amount. In terms of each unit of output data, \( P_i \), and \( \sum |M_i| \) values are calculated for all the basic set of output data.

Hence, following values are set to \( c_i \) coefficient:

\[
\begin{align*}
  c_{ij+1} &= c_{b0} \pm S_j, \\
  S_j &= 2\left|c_{ij} - c_{b0}\right|, \\
  s_i &= c_{b0},
\end{align*}
\]

where:

\[
\begin{align*}
  c_{b0} &= 0, \text{ then } S_1 = 1. 
\end{align*}
\]

\( c_{b0} \) is calculated. In this context, \(+\) sign is applied in Expression 6. \( \Delta M \) and \( \sum |M| \) values are calculated. Then \( c_{b1} \) value is determined. After that, \( \Delta M_i \) and \( \sum |M_i| \) are calculated according to \( c_{b2} \) value under \(-\) sign in Expression 6. As it follows from the subsequent, a predictive function may be discontinuous. Thus, the derived value interval is calculated within interval one and \( \sum |M_i| \) is studied whether extremum are available. If for instance, \( \sum |M_0| > \sum |M_0| > \sum |M_1| \) or on the contrary, it is supposed there are no extrema within the interval. \( \sum |M_i| \) value is defined. In this context, Expression 6 uses that very sign \(+\) or \(-\) which helped obtain the maximum value \( \sum |M_i| \). If \( \sum |M_i| > \sum |M_i| \), then further calculations with the use of the sign in Expression 6 are terminated it results from the fact that relying upon Expression 6, \( c_i \) and \( c_{b1} \) deviations from \( c_{b0} \) are symmetrical ones.

Further, Expression 6 is applied to identify new values \( c_{ij} \) and \( \sum |M_i| \) using \(+\) or \(-\), according to which the smallest \( \sum |M_i| \) value was obtained. It lasts until \( \sum |M_i| > \sum |M_i| < \sum |M_i| \) condition is met. Otherwise, if \( \sum |M_i| > \sum |M_i| > \sum |M_i| \) then the algorithm stops since it means that the considered interval includes extremum. If a computer has limited capabilities then we suppose it is in the interval between \( \sum |M_i| \) and \( \sum |M_i| \) values. Nevertheless, Expression 6 explains that change in \( c_{ij} \) doubles within each step. Thereby, the considered interval may involve more than one extremum. However, the calculated extremum may not match the actual one. It is required to find such an extremum corresponding to Condition 5. It is expedient to consider interval between \( c_{i_{j-1}} \) value, matching \( \sum |M_i| \), and the last \( c_{ij} \) value matching \( \sum |M_i| \) amount as two intervals. Interval one is between \( \sum |M_{i-1}| \) and a value marked as \( \sum |M_{i-1}| \) interval two is between \( \sum |M_{i-1}| \) and \( \sum |M_i| \). Consider extremum finding within interval one. The finding process is comparable within interval two.

Extremum is found using the modified technique of a halving argument division. The difference between terminal values of \( c_{i_{j-1}} \) and \( c_{i_{j+1}} \) interval is found; then it is divided into two. After that, the least is selected from \( c_{i_{j-1}} \) and \( c_{i_{j+1}} \) volumes. It is added by the absolute value of the determined difference. Intermediate \( c_{i_{int}} \) and corresponding \( \sum |M_{i_{int}}| \) have been identified. Similarly, midpoints of the obtained \( c_{i_{int}} \) and \( c_{i_{int+2}} \) intervals are determined as well as \( \sum |M_{i_{int}}| \) and \( \sum |M_{i_{int+2}}| \) values corresponding to them.

The smallest value is selected from the last ones. The interval with a larger value is excluded from calculations. The procedure is followed if \( \sum |M_{i-1}| > \sum |M_{i+1}| > \sum |M_{i+2}| > \sum |M_{i+3}| \) condition is met, and vice versa depending upon the computation order. In the context of precious calculation technique, \( \sum |M_{i-1}| > \sum |M_{i+1}| > \sum |M_{i+2}| > \sum |M_{i+3}| \) condition is always fulfilled. If any of intermediate \( \sum |M_{i+1}| \) values cannot match the abovementioned inequality then it is assumed that it divides \( \sum |M_{i+1}| \) value into two intervals, within which extremum is determined separately depending on corresponding \( c_{ij} \).

Each interval is studied until the difference between its terminal values become less than the specified accuracy \( \varepsilon \), \( \varepsilon \) may be considered as measuring accuracy of parameter \( P \) on the basic data set.

Of all the identified extrema, that one is accepted which correspondence to Criterion 5 is the greatest. Starting from Expression 6, the described algorithm was constructed relying upon \( \sum |M_i| > \sum |M_i| > \sum |M_i| \) inequality or vice versa. Criterion 5 defined the finding path. If the inequality could not be met then the abovementioned order is applied to search from \( \sum |M_i| \) value and, accordingly \( c_{b0} \) both towards greater \( c_i \) values and towards the smaller ones. Then Criterion 5 helps define the most expedient alternative.

In the context of the two mentioned techniques (i.e. a technique of index doubling and a halving division technique), calculations are controlled according to a criterion of uniformity of algebraic \( \sum |M_i| \) signs both during the current and previous step. If their \( \sum |M_i| \) signs differ then deviation of the calculated index from the actual \( \sum |M_i| \) indicator passed through its minimum and started increasing. In this case, an algorithm of double increase (decrease) stops and halving division algorithm starts for a local extremum determination within the interval of a varying coefficient identified with the help of a double increase (decrease) algorithm. The technique cannot take into consideration such an event when data vary symmetrically within each unit of the output data. Nevertheless, it is not important if Criterion 5 is involved.

Searching for a predictive function type may use an algorithm of double increase (decrease) of the desired coefficient in terms of all the available numerical coefficients; after that, halving division algorithm may start for each coefficient. The abovementioned will accelerate the algorithm on the whole; however, calculation accuracy may suffer.

Specify the described sequence of steps as algorithm 1. Describe algorithm 2 neglecting explanation of further construction of a predictive function during the stage.

Use of algorithm 1 makes numerical coefficients vary in turns. Such a calculation manner cannot give optimum results since decrease in \( \sum |M_i| \) may require simultaneous change in one and the same step of coefficient group. In this regard, their necessary changes may have different values as well as different directions. If one searches for direction and value of such changes with the help of simple iterations then minimum number of variants achieves \( A^n \), i.e. amount of arrangements of \( n \) indices in terms of \( n \). The procedure may complicate the calculations. Hence, it is required to select such coefficient groups which can vary as well as directions of the changes using another technique.
Assume that the predictive function takes the form in the context of which algorithm 1 cannot produce any results.

In this case, \( \Delta M_i \) values are calculated within each unit of the output data. After that, all the numerical coefficients, included by the predictive function, are multiplied by 1.5. Then, \( \Delta M_i \) values are determined again. Any of the coefficients recovers its initial value. \( \Delta M_i \) values are defined. That very coefficient, but being already a part of output value of the function, is multiplied individually by 1.5; \( \Delta M_i \) values are calculated. Following indicators are identified for each unit of the output data:

\[
\sum |\Delta M_i| = \left( |\Delta M_i - \Delta M_i| \right) + \left( |\Delta M_i - \Delta M_i| \right).
\]

The closer absolute value of the bracketed one and two differences is, the more stable changes in \( \Delta M_i \) are within the considered unit of the output data depending upon variations in one or another coefficient under changes in all other coefficients included in the predictive function. Theoretically, the two differences should be equal in the context of absolute value, and converse in terms of algebraic sign. It would mean that despite changes in a predictive function the analyzed coefficient gives the same variation in \( \Delta M_i \) in the context of its comparable change.

After the procedure, all coefficients of the predictive function are halved and each of the listed operations is repeated; numerical coefficient remains invariable. The halving and multiplication by 1.5 is used to make coefficient in cases one and two vary symmetrically to output value.

Then, \( \sum |\Delta M_i| \) values, derived in the cases one and two for each unit of output data during the coefficient consideration, are added. \( \sum |\Delta M_i| \) value is identified required for further use.

The procedures are performed for other coefficients included by the predictive function.

The listed operations are needed if only division by 1.5 and multiplication by 2 were not positive in terms of Criterion 5. If this is not the case then synchronous decrease or increase in the coefficients continues with simultaneous use of algorithm 1 for each coefficient.

Immediate search for optimum type of predictive function conforms to following order. The coefficient, according to which \( \sum (\sum |\Delta M_i|) \) is minimal, is considered first of all. It is halved and all other coefficients are multiplied by 1.5. Inverse operation follows. Then it is divided by 2 and multiplied by 1.5 from similar value of the function. After that it lets alone; operations are performed with other coefficients. \( \sum |\Delta M_i| \) is calculated according to (4). Coefficient one can use \( \Delta M_i \) values identified while determining \( \sum (\sum |\Delta M_i|) \) for it.

(5) is applied to select the most appropriate variant. In such a way, direction of changes in the coefficient is determined as well as in the set of others.

Next, algorithm 1 is brought into action taking into consideration the determined directions of changes. In this case, the predictive function is searched from a value being the initial rate for the circumstances. Simultaneously, the selected coefficient varies as well as the set of other coefficients. If the procedure is not successful in terms of Criterion 5 then following variant is selected according to the expediency.

After searching through the listed calculation alternatives, one proceeds to a coefficient from the increasing number of \( \sum (\sum |\Delta M_i|) \). The calculation nature depends upon the fact that if previous coefficient where \( \sum (\sum |\Delta M_i|) = \text{min} \) demonstrates positive results according to (5) then there is high probability that it will remain while defining following coefficients, i.e. a tendency of its required variation will not be altered.

In the context of each of the coefficients (exclusive of the first one in the calculation procedure), the procedure of search for the change tendency should involve consideration of such alternatives when all coefficients experience either increase or decrease if the previous one varied.

If all the coefficients have been examined with no positive results then the process is repeated in reverse, i.e. the coefficient where \( \sum (\sum |\Delta M_i|) = \text{max} \) is the first one to be considered. If no results are obtained then algorithm 1 is not applied for the calculations; however, the search for variations in the coefficient direction reminds a quantity being undesirable but the most adequate according to (5). It starts the computational procedure. As a last resort, synchronized variation in the same path of all the coefficients may be assumed in terms of which one or another amount gives the best result while using algorithm 1. If no one of the listed variants is expedient then algorithm 2 stops. It is possible to enumerate many more computational procedures which are senseless for the paper.

Describe a process of predictive function construction using algorithms 1 and 2. As it has been mentioned above, the process starts from \( c_1 \) coefficient according to algorithm 1. After its optimum value has been defined, search for its equivalent coefficient at the \( p \) degree in terms of \( x_1 \) (3) initiates since the degree change gives the most sensible results. Next step is iteration with the first \( x_1 \). It should be done until results are obtained. Then a new value of \( c_1 \) coefficients starts being searched for. It should be done since the calculation procedure may achieve some specific computational accuracy of \( P_{x_1} \) values known from the experience. Following calculations may stop. If \( c_1 \) search is effective then its analogous values in the degrees under both \( x_1 \) are defined again. If the procedure is not resultative then algorithm 2 starts operating. If it is resultative then algorithm 1 is used as it has been mentioned earlier. The next step is search for \( e_1 \) modifier (2); the process is repeated. If no result is obtained then second bracket is added to Expression 3; the process is repeated starting from search for \( c_1 \) coefficients under \( x_2 \), and then from \( d_1 \) and \( b_1 \) coefficients. Round brackets are added to Expression 3 until all \( x_1, x_2, ..., x_n \) values are applied known within the basic output data set. After that search for \( c_1 \) starts in the degrees with round brackets from the brackets which gave the best result.

It is assumed in the calculation procedure that finding an even degree root from a negative number may be 0; 1; and \(-1\) value; the same being if a number value is positive; negative number value; and negative root value. It is assumed that fractional degree finding from zero may be equal to 0; 1; and \(-1\). The abovementioned describe discontinuous dependence between the indicators.

Two similar arguments in round Brackets 3 with similar indices, for instance \( \left( a_{11}^p x_1^p + b_{11}^p x_1^p \right)^p \), help the predictive function become very flexible since they reduce to different \( p \) degrees like \( a \) and \( b \) coefficients. It is especially important while giving discontinuity characteristics to a predictive function since owing to \( a \) and \( b \) coefficients in terms of certain \( x_1 \) and \( x_2 \) values, differing within the output data units, the expression in brackets may become of zero or negative value. Taking into consideration \( p \) degree behind the brack-
ets, the expression in brackets may be disintegrated into any quantity of variables thus making it possible to apply (1)-(3) function to describe any mathematical dependence.

Using all available coefficients of a predictive function, optimum values are defined with the help of algorithms 1 and 2 in succession. In the context of algorithm 1, the search starts from the coefficients which were the most successful in the process of previous calculations. Then round brackets are added to (3) with $a_{21}, b_{21}, ... , a_{mn}$ and $b_{mn}$ coefficients. In this context, the new $x_1, x_2, ... , x_n$ coefficients are those ones which demonstrated the best results in the process of previous calculations. After that, (3) searches for types of functions of $p$ degrees under $x_1, x_2, ... , x_n$ as well as numerical coefficients in accordance with the abovementioned procedure. Increase in the order of $p$ degrees is unlimited.

If $f_i (x_1, x_2, ... , x_n)$ value, determined in such a way, is result then construction of (1) starts to identify modifiers $h$ and $g$. In the same way, (3) is added by all $F_i (x_1, x_2, ... , x_n)$ values and rates of $p$ degrees are defined. In general, predictive function is constructed increasing its complexity and unwildness with constant adaptation of earlier calculated coefficients in terms of prevlance criterion of coefficients from algorithm 1 demonstrated the best results in the process of earlier calculations.

If the abovementioned ways could not improve the predictive function or the improvement turned out to be minor (i.e. it was improved by a value being less than $\varepsilon$ rate described in characterization of halving subalgorithm), it is time to proceed to algorithm 3.

In the context of the available predictive function, each unit of output data involves algebraic $\Delta M_{1, \alpha 1}$ values. They are ranked depending upon their increase; then the closest of them are grouped in pairs. The grouped units of the output data may include quite different $x_1, x_2, ... , x_n$. For instance, their spatial distancing may be great. Hence, the grouping follows a criterion of the peak values of indices being measured.

After each pair grouping starting from minimum $\Delta M_{1, \alpha}$ values, the predictive function is searched separately. The procedure is based upon previously derived predictive function if it gave positive results according to (5). If not, the finding process starts again.

In this context, the function, being analyzed within any unit of the output data, is added by a set of variable output data included by its paired unit. However, the function type for both units of the output data will be similar. The values within the paired unit are just put into a corresponding place of the predictive function while calculating the output data within the unit. If one assumes at least any differences in this part of a predictive function, involving variables being foreign for the output data units, then the function will transform into two numerical equal to $P_{lm}$ coefficients. Such an addition of variables makes it possible to assess dependence between $P_{lm}$ values within the output data unit as well as between the values of variables in the output data unit which is the closest to this one relative to the general predictive function in terms of maximums of its values relying upon the grouping type.

After that, the group moves one step up or down depending upon $\Delta M_{1, \alpha}$ ranking. The grouping repeats. The grouping variant is selected as the most relevant one from the viewpoint of Criterion 5. Then for each pair individually $\Delta M_{1, \alpha}$ a dependence upon values of variables paired by means of algorithms 1 and 2 is derived. It should be done to find a sequence of similar grouping of the output data units within the modeled data set where $P_{lm}$ values are not available rather than introduce modifications. The matter is that the modeled set of the output data may include such output data quantity differing from the basic ones, and follow the dependence between indicators calculated within the basic set only in the form of a ratio rather than a ready addition.

A function of $\Delta M_{1, \alpha}$ dependence upon the values of variables, being a part of a pair, involves all known values of the pair inclusive of $P_1$ and exclusive of $P_m$.

After the grouping, new $\Delta M_{1, \alpha 2}$ values originate within the basic output data set. Stage two of the grouping starts. In turn, $\Delta M_{1, \alpha 2}$ rank in the increasing order. (4) helps calculate $\sum | \Delta M | $ value obtained during previous grouping step on the basic data set. The output data units are grouped depending upon the closest $\Delta M_{1, \alpha 2}$ above the values. The paper will not consider a potential for the output data set to group between themselves together with $\Delta M_{1, \alpha 1}$ and $\Delta M_{1, \alpha 2}$ criteria; however, it is quite a possible process.

After the data grouping in terms of $\Delta M_{1, \alpha 2}$ criterion, two computational scenarios are probable. Scenario one is as follows. An output data unit is subtracted from a group derived during stage one to be added to another group. Scenario two is fusion of two groups into one. The best option is selected using Criterion 5. All the data, used for the transferring unit of the output data, are saved added by $\Delta M_{1, \alpha 1}$ value. The output predictive function is that one which satisfied (5) mostly at the previous stage. The function is added by variables; they are replaced by the data of neighboring units.

During stage two $\sum | \Delta M | $ may increase since predictive functions become of more general nature. In such a case, grouping lasts until minimum accuracy, determined experimentally, is obtained. It takes place because the less group number is within the basic set the easier it is to work with a set being modeled; nevertheless, that can be determined only in practice. The derived dependencies are substituted to the modeled set as follows.

The dependence, obtained within the basic set, between the first by order minimum $\Delta M_{1, \alpha 1}$ value and its group variables during the initial grouping stage is substituted to the modeled set in any two units of output data. Then, one of the units lets alone and another unit is substituted by the following one etc. In this manner, a group is selected matching mostly the dependence obtained within the basic set between $\Delta M_{1, \alpha 1}$ values and variables from the group. The derived unit of the output data within the modeled set lets alone; the first one is substituted by another which demonstrated the best result in the previous case while grouping from the first. If the result is worse than the previous one, the grouping process stops. At the stage, the group is considered as the finished one. Similarly, all other groups are determined during stage one. If the modeled data set has more units of the output data than the basic one then in turn the obtained groups are formed in such a way to be less in number to compare with the basic set. The difference between $\Delta M_{1, \alpha}$ maximum and minimum is calculated relying upon the values obtained within the set being modeled while grouping. After that, it is divided into the number of groups obtained within the basic set during stage one. According to the interval, grouping takes place inside each period. Groups, obtained during stage one, are divided into two subgroups following a principle of
symmetry from inside. In the context of the subgroups, all indicators are averaged. In accordance with the abovementioned, the second grouping stage starts.

The expected predictive accuracy is within the basic set at the last grouping stage for each group individually expressed in the form of dependence upon indicators included by the group.

3. Results and discussion

Geometrization and prediction are based upon geological prospecting of a deposit.

One of the basic objectives of geological prospecting is description of arrangement regularities of deposit parameters (i.e. its foot and floor, thickness, content of useful and harmful components etc.). In this context, prospecting should result in the required accuracy under less sampling points.

Currently, iteration prospecting strategy is expedient. To obtain information on the geometry of deposit occurrence, the first prospecting stage is well drilling at large distances from each other. If prospecting purposes (i.e. accuracy of reserve determination and occurrence geometrization) are not achieved then the number of wells increases (i.e. average interval decreases). The prospecting results are assessed, a problem of further network densification is solved etc.

While transitioning from one prospecting stage to another, the interval (i.e. density of the exploration network) usually varies intermittently. Hence, it is required to assess accuracy at each stage. Selection of rational density of exploration network at one or another stage originates a reverse mission of interval determination according to the necessary accuracy.

An exploration error consists of two components: representativeness error and interpolation error.

Representativeness error $\sigma_R^2(l)$ evaluates deviations of actual parameter variation along a profile line from the interpolated value between two measurement points.

To evaluate $\sigma_R^2(l)$, following expression is often used:

$$\sigma_R^2(l) = \frac{1}{C_k} M \left( \Delta k^2 \right),$$

where:

$$C_k = \frac{(2K)!}{K'!} \cdot \Delta l = x_0 - x_1, \Delta_2 = x_0 - 2x_1 + x_2 \text{ etc.}$$ and $x_0, x_1, x_2$ are measured values of $Y_p$ parameter along a profile line with $l$ interval; and $M$ is mathematical expectation value.

Use of Formula 8 raises a problem concerning K order of successive differences. K value selection depends upon a degree of algebraic polynom approximating the spacing. If one applies $r$ order polynom to approximate the spacing then $K = r + L$.

Interpolation error $\sigma_I^2(l)$ assesses deviations of actual values of $Y$ parameter from an approximate $Y_l$ values obtained by means of linear interpolation right between two points of the parameter measurement:

$$\sigma_I^2(l) = M \left( Y - Y_l \right)^2.$$ (9)

Prospecting error depends heavily upon the nature of arrangement of deposit parameters.

Spacing of any deposit parameter either in the direction of a plane or within it can be represented by means of a curve or a surface of more or less complex vibratory nature. Analysis of such a curve or surface distinguishes natural, objective, and subjective variability components which grouping shows the observed variability. Natural variability is connected with observation methods; it is determined by means of the deposit genesis. The observed variability is a footprint of the natural variability in perception through the observation outcomes. At following stages, it takes a form of mathematical or geometrical model generalizing the prospecting results. Along with the exploration model densification, subjective perception of the deposit parameter spacing approaches the objective one. However, their complete coincidence is impossible. Hence, deposit prospecting always involves certain degree of uncertainty as for the information on the regularities of spacing of one or another parameter either increasing or decreasing with the increased well number and shortening distance between them. There comes a point when observation results of a parameter start demonstrating regularity and a tendency originates. The exploration interval, in terms of which it happens, is a critical period or correlation radius $l_c$.

To define numerical value $l_c$, the normalized autocorrelation function is applied, which certain values are determined using the formula:

$$\rho_x(l) = \frac{1}{\sigma^2(N-k)} \sum_{i=1}^{N-k} \left( u_i - \bar{u} \right) \left( u_{i+k} - \bar{u} \right),$$ (10)

where:

$$\bar{u} = \frac{1}{n} \sum u_i \text{ is arithmetic average of observation series of the parameter; } \sigma^2 \text{ is dispersion of the series; } k = 1, \ldots, N - 1 \text{ is exploration interval; and } N \text{ is total number of exploration steps along the transverse section.}$$

The first value of the normalized autocorrelation coefficient, differing from zero in the process of the interval decreasing from $N$ down to $0$, denotes the critical interval value.

The observed variability is divided into the random and regular ones. Separation of the types is based upon both interpretation of philosophical categories of random and regular, and upon mathematical criteria.

Boundary of randomness (uncertainty) measure is a value of standard $\sigma$ of the analyzed parameter rates. If exploration intervals exceed $l_c$ then the regularity cannot be demonstrated. If exploration intervals are less than critical ($0 < l < l_c$) then the regular and random components are in the certain ratio in the parameter spacing. It should be mentioned that in terms of even rather small test intervals $l_0$ the spacing involves random component $\sigma_0$ due to technical measuring errors depending upon the measurement type, mineral, sample geometry etc.

Consequently, the regular (the coordinated and spatially correlative) variability is that one being characterized by smooth shift of a feature as well as stability of its increasing sign; in addition, it provides the ability to identify the feature value within the intermediate points between the observation points. Random variability is that one being characterized by nonavailability of any dependence between the feature values within two different points upon distances between the points as well as upon the distance to any other point which may be assumed as a datum point (origin of coordinates).

Despite the significant diversity of mathematical criteria separating random component from the general variability assessment, the problem of qualitative variability evaluation while its dividing into random and regular is far from its satisfactory solution.
For \( C_3 \) sample, differing in the maximum variability, distance from a block doubles becoming \( 2d \); for \( C_2 \) sample, differing in minimum variability, distance from a block is \( d \); and for \( C_5 \), being of an average variability level, the distance is \( d = [(x_0 + 2(y_0)^2)^{1/2}] \).

Hence, average \( C(\%) \) content is:

\[
C = \frac{C_1 \cdot (2d_1)^{-2} + C_2 \cdot (2d_2)^{-2} + C_3 \cdot (x_0^2 + 2y_0^2)^{-1}}{(2d_1)^{-2} + (2d_2)^{-2} + (x_0^2 + 2y_0^2)^{-1}}.
\]

(15)

Inverse distance methods are applied only for small regular shape blocks.

Method of autocorrelation functions is applied for deposits with a simple form if thickness is uniform and viability of grade indices is minor. The method idea is as follows. According to well test data in the \( i^{th} \) direction, the normalized function (i.e. correlation function) is constructed which value is determined relying upon its indicator variability as well as difference of distances between the test points:

\[
\rho(l) = \sum \frac{s^0 (l_i) \cdot s^0 (l_{i+1})}{(n-1) \cdot D_n},
\]

(16)

where:
- \( s^0 (l_i) \) – the centered value of the indicator;
- \( l_0 \) – distance between neighboring numbers of the series;
- \( n \) and \( D_n \) – number and dispersion of the output data of the test;
- \( l \) – distance between the series numbers being analyzed \((l = \text{dist} ; t = 1, 2, 3, ..., m \leq n)\).

The normalized autocorrelation function is approximated in such a way:

\[
\rho(l) = k e^{-c \alpha \beta} \cos \beta \ell + c = k g \cos \beta \ell = c.
\]

(17)

Like in a net method, a block is networked. Indices within the network nodes are determined while weighing indicators in wells that surround the node. \( g = e^{-c} \) component is applied as the weight.

The interpolated value of an index in the \( i^{th} \) node is identified following the Formula:

\[
z_j = \frac{\sum_{i=1}^{n} g_i z_i}{\sum_{i=1}^{n} g_i},
\]

(18)

where:
- \( g_i \) and \( z_i \) – exponential components and values of the indices within the neighboring wells.

Geostatic method is the most accurate technique. Kriging is another name.

Geostatic theory differs in:

1. explicit mathematical definition;
2. possible analytical form of all calculations as well as high degree to unify programs for data processing machines;
3. while calculating reserves, average values of grade indices are considered as functions of block geometry; arrangement of the initial test data; and spatial variety of geometrical characteristics are also involved. On the other hand, anisotropy, being a varying mineralization degree, is also taken into consideration.

Kriging solves two key problems:

1. determination of ore reserves;
2. identification of the assessment accuracy.
Average content in a certain block is defined using the Formula:
\[ \hat{z} = \frac{1}{n} \sum_{i=1}^{n} a_i \cdot z(x_i), \quad (19) \]

where:
- \( z(x_i) \) – mineral content in the samples; %;
- \( a_i \) – is weighing (kriging) coefficient.
- \( a \) – coefficient is determined while solving a system of kriging equations. \( a \) value depends upon the grade characteristics of content variety within the amount of the considered ore body as well as upon the block belonging. In this context, the basic purpose is to find such weighing coefficients owing to which the most adequate assessment may be achieved in addition to the least assessment error.

There are several kriging modifications which selection depends upon numerous factors: exploration system, mining system, block geometry, mutual arrangement of the analyzed block, and test data involved in the assessment.

Pivdgzk JSC open pit applied the developed mining and geometrical predictive method. The exploded rock mass was tested during exploration within the irregular shape areas as well as different cross-section dimensions. The detailed exploration wells within the site were drilled through irregular test at 50-200 m distances. Multidimensional prediction method as well as kriging has been applied to construct predictive function of useful mineral content and isoline of prognostic content. The degree of confidence to the results has been defined as that one inversely related to a distance from the centers of sites within which the exploded rock mass from the neighboring detailed exploration sites has been tested since the value had the greatest influence on the interpolation error. As a result, the most accurate outcomes had greater priority while assessing accuracy of the constructed predictive function.

According to the known data, functional predictive dependencies have been identified within the mined-out sites. Figures 1-3 demonstrate them.

Consequently, functional dependencies between magnetic ore in the exploded rock mass and iron content in the context of deep exploration wells have been defined since among all algorithms, involved in the decision procedure, numerical coefficients have been indentified only for the values. The activities have helped group deposit sites according to the derived dependence; predictive results have been interpolated towards nodes of 50×50 m square network at a scale of 1:2000 and a cross-section height of 0.5% content.

According to the predictive procedure, square grid was used to define the forecast value of iron content connected with magnetite in the blasted rock mass. Kriging was used to interpolate the values which became the basis for long-term planning. The interpolation has helped construct isoline plan of predictive magnetic iron content (Figs. 4-6).

Predictive data for the current planning were produced while specifying the obtained e-model in accordance with operational exploration within the points which spatial location was determined relying upon production needs, and had irregular nature. The graphic and analytical prediction results were applied to solve mining and geometrical problem to plan extraction operations in the mode of the averaged content of a useful component, i.e. determination of optimum tendency and efficiency of mining relying upon both technological and economic indicators of the enterprise.

In the context of the developed methods for long-term planning, relative error of the forecast concerning magnetic ore content is not more than 6.8%. Owing to the improved forecast, used as the basis for more accurate mining scheduling, average ore loss coefficient within the deposit sites decreased by 0.03%; as for the ore dilution coefficient, its average decrease coefficient is also 0.03.

As the study has shown, the predictive methods are quite efficient and applicable. The findings enable both current and long-term planning which improves efficiency of mining operations. The techniques, involved by the methods, expressed themselves positively. The abovementioned helps draw a conclusion on the necessity to improve and advance the techniques.
Figure 4. Isolines of predictive content of magnetic iron within 165-180 m level in Pivdgzk JSC open pit

Figure 5. Isolines of predictive content of magnetic iron within 180-195 m level in Pivdgzk JSC open pit

Figure 6. Isolines of predictive content of magnetic iron within 165-180 m level in Pivdgzk JSC open pit

4. Conclusions

Findings of the research have helped obtain regularities to improve the predictive efficiency to arrange grade indicators of iron-ore deposits. For the purpose, a multidimensional heuristic prediction technique was applied basing upon a random degree polynomial as well as geostatic methods providing significant increase in the forecast accuracy, and efficient use of the available surveying and geological data owing to quantification of qualitative genetic relationships between indicators.

It is expedient to assume the indices coinciding in distribution law with the forecasted one as the arguments of the random degree polynomial as those ones having more probable regularity with spacing of the predicted component of a mineral.

The assessment of output surveying and geological data, obtained on the basis of irregular testing network, is supported by kriging method being the most accurate among all known interpolation techniques. It is expedient to develop mining and geological deposit model relying upon the use of multidimensional random geochemical field.

On the whole, the developed predictive methods are efficient while matching mining needs. They help assess mineral reserves to improve significantly mining scheduling. Geostatic methods make it possible to evaluate and process output geological data. The developed self-organized predictive algorithm is flexible; in such a way, it can be applied under different mining and geological conditions to plan and evaluate various mining techniques. Relying upon the results, the self-organized and geostatic methods are very promising tendency for future studies. The methods need further development and perfection to improve their efficiency and use in the context of both ore and non-metallic minerals.
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References
Прогнозна геометризація якісних показників залізорудного родовища

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Мета. Розробка методу прогнозування показників залізорудних родовищ на основі ідентифікації основних факторів та створення нового алгоритму геометризації, а також визначення найбільш прийнятного способу оцінки геологічних даних як основи для геометризації та прогнозування.

Методика. Створення алгоритму прогнозування, що самоорганізується на основі сукупності відомих методів і розробки нових математичних моделей. Розгляд різних способів оцінки даних геологічної розвідки, оцінка їхньої ефективності в умовах залізорудного родовища та вибір найбільш ефективного способу. Використання геостатистичних методів дає можливість оцінки та обробки виходної геологічної інформації.

Результати. Отримано залежність вмісту магнетитового заліза в кар’єрі ПівдГЗК від геолого-технологічних факторів. Використано геометризацію родовища та отримано прогнозну гірничо-геометричну модель ділянки родовища. Виявлена залежність від геолого-технологічних факторів, що дозволяє оцінювати запаси корисних копалин гірничого підприємства.

Наукова новизна. Розроблено метод прогнозування гірничо-геологічних показників залізорудного родовища на основі алгоритму, що самоорганізується. Виявлено залежність від геолого-технологічних факторів, що дозволяє оцінювати запаси корисних копалин гірничого підприємства.

Практична значимість. Розроблено методику геометризації залізорудних родовищ, що дозволяє з високою точністю планувати гірничі роботи, а також підписати їх ефективність. Розроблений алгоритм прогнозування, що самоорганізується, є гнучким у застосуванні та може бути використаний у різних гірничо-геологічних умовах для забезпечення планування та оцінки різних технологій ведення гірничих робіт. Самоорганізовані методи та геостатистичні методи оцінки є досить перспективним напрямом дослідження.

Ключові слова: алгоритм що самоорганізується, геостатистичні методи, запаси, геометризація, якісні показники, прогнозування.