

## ENSURING THE SPECIFIED POSITION OF MULTISUPPORT ROTATING UNITS WHEN DRESSING MINERAL RESOURCES

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### ABSTRACT

**Purpose.** The solution of the problem for determining a specified vertical displacement of supports in a rotating unit, taking into account the yielding in the crosscut of the supporting node, as well as the peculiarities of the structural design.

**Methods.** When performing the work, the approaches based on the elasticity theory principles are used, taking into account the variable transverse crosscut of the unit and the different high-altitude positions of supports. To determine the general expressions describing the axis position of the studied multisupport structure, the Cauchy' function method was used.

**Findings.** A system of solvable algebraic equations has been obtained that enables to perform calculations for a beam-type cylindrical structure on rigid and elastic supports. The expressions are presented for determining the value of vertical displacement of the supports, taking into account the operational characteristics of a unit, in particular the presence of the influence of neighboring supports and the value of existing loads. The cases have been studied of implementing the adjusting displacements, using the obtained equation system with the complete unloading of the supporting nodes, as well as with the use of the technical diagnostics data – the determined values of the total displacements of the supports. Expressions have been obtained for calculating the desired adjustment parameters. An algorithm is proposed for performing the computing operations.

**Originality.** The method of adjustment of the specified position of supports in a rotating unit has been further developed, which is carried out through determining the elastic components, rigid displacements, complete displacement in the supporting node, as well as the results of technical diagnostics.

**Practical implications.** The calculations have been made taking into account the yielding of the supports in the range of maximum and minimum values. It is shown that the value of certain supports displacement, which should be taken into account as a result of their yielding, can reach the extremum standard values. The presence of previous displacements of the supports predetermines a significant acting forces redistribution in the rotating furnace body and is determined by their direction. The value of adjusting displacements has been determined, which should be performed to obtain a rectilinear axis of rotation. It has been revealed that it is appropriate to implement a parallel displacement of the projected axis of rotation for reducing the adjustment parameters in the supporting nodes.

**Keywords:** rotating units, beam-type structures, supports adjustment, elastic deformations of supports, multisupport units, axis of rotation

### 1. INTRODUCTION

In the mining and metallurgical industries, when mining, transportation, dressing and processing of bulk materials, the broaching, cylindrical units are widely used that are located on separate supports. These include conveyors, mills, screens, drying drums, furnaces and other rotating units for various technological purposes. In this case, the misalignment of a linear part, in particular

the axis of the structure rotation, from a specified vertical position leads to exceeding the design loads and the occurrence of a complex stress-strain state (Bilobran & Kinash, 2002). As a result, the local destruction of material is often observed in the form of increased rate of corrosion (Andreikiv, Skal's'kyi, Dolins'ka, & Dzyubyk, 2018), wear of moving parts (Dreus, Sudakov, Kozhevnikov, & Vahalin, 2016), as well as formation and development of cracks (Andreikiv, Dolins'ka, Lysyk,

& Sas, 2017), especially in areas of concentration of technological stresses of the type, in particular welded joints (Dreus, Kozhevnikov, Lysenko, & Sudakov, 2016; Dziubyk, Nykolyshyn, & Porokhovs'kyi, 2016).

The typical reasons for the supports displacement, especially in the case of multisupport rotating units, should include a significant mass of the structure, the impact of locally applied forces in the supports, wear of the working surfaces (Trach, 2007; Kovalev, et al., 2009; Maksymovych, Lavrenchuk, & Solyar, 2019), cyclic alternate loads, location in areas with unstable soils, as well as elastic deformation of the supporting node elements with significant operational loads (Kuzo, Mykolskiy, & Shevchenko, 1982; Dziubyk, Kuzo, & Prokopyshyn, 2009). In this regard, it is necessary to perform periodic additional adjustments of the supports position in the production conditions (Kuzo, Mykolskiy, & Shevchenko, 1982; Kuzo & Dziubyk, 2009). In such a case, it is important to know the value of the supports displacement, taking into account the possible external influences and peculiarities of the structural design of the rotating unit.

*Analysis of existing sources.* At the present time, various calculation methods are widely used, based on the elasticity theory principles and the use of computer simulation methods (Kuzo, Mykolskiy, & Shevchenko, 1982; Kuzo & Dziubyk, 2008; Kuzo & Dziubyk, 2009; Kononenko, Khomenko, Sudakov, Drobot, & Lkhagva, 2016; Khomenko, Kononenko, Myronova, & Sudakov, 2018). The finite element method, which ensures the visibility of the results and the possibility to take into account the geometry features is the most characteristic one (Polivanov, Belov, & Morozova, 2017). At the same time, in the case of multisupport broaching structures, it is rather difficult to perform such simulation and unify calculations. More acceptable is to present the considered units according to the model of continuous beam (Timoshenko & Woinowsky-Krieger, 1959; Kuzo, Mykolskiy, & Shevchenko, 1982). The most studied here is the case of the beam with constant and variable transverse crosscut on rigid supports (Kuzo, Mykolskiy, & Shevchenko, 1982; Kuzo, & Dziubyk, 2009). Also, the influence was shown of the rigidity coefficient of the support elements onto the value of displacements in the supporting node as a whole (Kuzo & Dziubyk, 2008). The strength characteristics of the beam on elastic supports have been studied separately (Kuzo & Dziubyk, 2007; Kuzo, & Dziubyk, 2009). However, a comprehensive assessment of the beam structure strength of a rotating unit with possible vertical displacements, in the presence of elastic deformations of the supports and available information about their relative high-altitude position, has been studied insufficiently.

*Setting of the problem.* When performing calculations, it is expedient to consider the available results of technical diagnostics and testing of rotating units both after stopping and when running into commercial operation. A clear algorithm should also be developed for implementing the procedure of calculating the value of adjusting displacements. Therefore, it is relevant and important to study the model of static equilibrium of a beam with variable rigidity on elastic supports in case of preliminary set displacement, as well as the ability to perform its correction.

## 2. METHODS

Given the structural design of the multisupport unit, it is expedient to present it in the form of some beam structure. It has a certain overall length ( $l$ ), but at the same time piecewise constant flexural rigidity. The latter one is conditioned by various transverse crosscuts of the studied units to ensure the special technological operating conditions. The studied beam lies freely on elastic supports ( $N_R$ ) under the action of applied concentrated and distributed piecewise constant loads (Fig. 1).

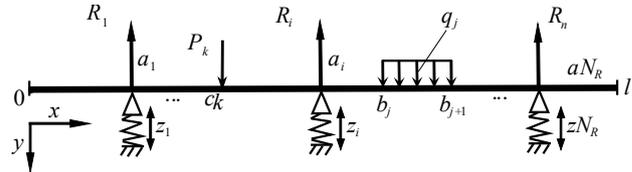


Figure 1. Diagram of a multisupport unit in the form of a beam structure with piecewise constant bending rigidity on elastic supports (Dziubyk, Kuzo, & Prokopyshyn, 2009)

The following designations are introduced here for the diagram in Figure 1:  $a_i, k_i, z_i, i = 1, N_R$  – coordinates, yielding coefficients and initial displacements of separate supports; ( $d_m, d_{m+1}$ ),  $m = 1, N_D$  – areas of the unit where the bending rigidity  $D_m$  of the beam structure is constant. At points  $x = c_k, k = 1, N_p$ , the concentrated forces  $P_k$ , such as a crown drive gear, a bandage and the like, act on the beam structure. Also, the presence of a protective lining, chain loops, etc., are taken into account. For this purpose, in separate areas ( $b_j, b_{j+1}$ ),  $j = 1, N_q$  (within the area of constant rigidity), the distributed forces of constant intensity  $q_j$  are applied. It is taken into account here that  $\Delta_j = b_{j+1} - b_j$  is the length of the area with constant load,  $b_j^0 = (b_{j+1} + b_j) / 2$  – is their centers. External load causes the reaction of supports  $R_i, i = 1, N_R$ .

The deflection of the considered beam  $w(x)$  satisfies the following differential equation (Timoshenko & Woinowsky-Krieger, 1959; Dziubyk, Kuzo, & Prokopyshyn, 2009; Westergaard, 2014):

$$(D(x)w''') = q(x), \quad x \in (0, l), \quad (1)$$

where:

$$q(x) = \sum_{i=1}^{N_R} R_i \delta(x - a_i) + \sum_{k=1}^{N_p} P_k \delta(x - c_k) + \sum_{j=1}^{N_q} q_j [\theta(x - b_j) - \theta(x - b_{j+1})] \quad \text{external load;}$$

$\delta(x - \alpha)$  – Dirac delta function;

$$\theta(x - \alpha) = \begin{cases} 0, & x < \alpha \\ 1, & x \geq \alpha \end{cases} \quad \text{– Heaviside function.}$$

The bending moment and shear force in the considered beam are determined from its displacements:  $M(x) = -D(x)w''(x)$ ;  $Q(x) = M'(x)$ .

Since the beginning and end of the rotating unit are free, the beam edges should be considered in a similar way: the conditions of equality to zero of bending moments and shear force are satisfied on them  $M(0) = 0$ ;  $Q(0) = 0$ ;  $M(l) = 0$ ;  $Q(l) = 0$ .

Each support is exposed to a force, opposite to the support reaction  $R_i$ , which causes the elastic displacement of support ( $\delta_i = -\kappa_i R_i, i = 1, N_R$ ).

From the condition of ideal beam contact with the support, the equality is obtained, which determines the complete displacement in the supporting node:

$$w(a_i) = -\kappa_i R_i + z_i, i = 1, N_R. \quad (2)$$

Having used the Cauchy' function method (Zoriy, 1979; Hashchuk & Zorii, 1999; Jaroszewicz & Zorij, 1999), to construct the general solutions of differential equations with variable coefficients (Gashchuk & Zorii, 2002), the general solution of equation (1) can be written as follows:

$$M(x) = -\sum_{i=1}^{N_R} R_i \theta(x-a_i)(x-a_i) - \sum_{k=1}^{N_P} P_k \theta(x-c_k)(x-c_k) - \sum_{j=1}^{N_q} q_j \begin{cases} 0, & x < b_j \\ (x-b_j)^2 / 2, & b_j \leq x < b_{j+1} \\ \Delta_j (x-b_j^0), & b_{j+1} \leq x \end{cases}; \quad (4)$$

$$Q(x) = -\sum_{i=1}^{N_R} R_i \theta(x-a_i) - \sum_{k=1}^{N_P} P_k \theta(x-c_k) - \sum_{j=1}^{N_q} q_j \begin{cases} 0, & x < b_j \\ x-b_j, & b_j \leq x < b_{j+1} \\ \Delta_j, & b_{j+1} \leq x \end{cases}.$$

The next stage is determining the force factors acting on the body of a multisupport rotating unit. As a result, they cause deformation of the lengthy structure of the body and reduce its durability. Therefore, it is also important to develop an algorithm for adjusting the high-altitude position of the supporting nodes for obtaining the optimal position of the axis of rotation.

### 3. RESULTS AND DISCUSSION

The beam deflection, bending moment and shear force are fully expressed through the initial displacements  $w_0$  and the angle of rotation  $\theta_0$ , the reaction of the supports  $R_i, i = 1, N_R$  and the external load. From the boundary conditions, at  $x = 1$ , the following equations (5) can be obtained:

$$\sum_{i=1}^{N_R} R_i (l-a_i) + M_\Sigma = 0; \quad (5)$$

$$\sum_{i=1}^{N_R} R_i + P_\Sigma = 0.$$

The equations (5) are the conditions for the equilibrium of moments and transverse forces relative to the right end of the beam and the condition of the transverse forces equilibrium.

From the conditions of ideal contact between the beam and the supports (2), the  $N_R$  is obtained of the equation (6):

$$w_0 + a_i \theta_0 + \sum_{m=1}^{N_R} \psi_{im} R_m + \kappa_i R_i = z_i + \chi_i. \quad (6)$$

Equations (5), (6) constitute a system of  $N_R + 2$  linear algebraic equations for determining  $N_R + 2$  of unknown  $w_0, \theta_0$  and  $R_i, i = \overline{1, N_R}$ . The following designations

are introduced here:  $P_\Sigma = \sum_{k=1}^{N_P} P_k + \sum_{j=1}^{N_q} q_j \Delta_j$  – the total

$$w(x) = w_0 + \theta_0 x + M_0 \frac{\partial}{\partial \alpha} K(x, 0) + Q_0 K(x, 0) + w_*(x, 0), \quad (3)$$

where:

$$K(x, \alpha) = \int_{\alpha}^x \frac{(x-s)(s-\alpha)}{D(s)} ds \text{ – is the Cauchy' function;}$$

$$w_*(x, \alpha) = \int_{\alpha}^x K(x, t) q(t) dt.$$

Here, the initial parameters  $w_0, \theta_0, M_0, Q_0$  correspond to the deflection, angle of rotation, bending moment and shear force at the left end of the beam structure ( $x = 0$ ). The bending moments and shear forces are further found by the formulas in the following form:

transverse force from the specified loads;

$$M_\Sigma = \sum_{k=1}^{N_P} P_k (l-c_k) + \sum_{j=1}^{N_q} q_j \Delta_j (l-b_j^0) \text{ – the total moment}$$

of the specified loads relative to the right end of the beam;

$$\chi_i = -\sum_{k=1}^{N_P} P_k \theta(a_i - c_k) K(a_i, c_k) + \sum_{j=1}^{N_q} q_j H_j(a_j) \text{ and}$$

$$\psi_{im} = \theta(a_i - a_m) K(a_i, a_m).$$

Let us show that an additional linear displacement of the supports:

$$\Delta z_i = \alpha + \beta a_i, i = \overline{1, N_R}, \quad (7)$$

where:

$\alpha, \beta$  – are constant, does not change the reactions of supports.

Thus, having substituted the expression (7) in equation (6) we obtain:

$$w_0 - \alpha + a_i (\theta_0 - \beta) + \sum_{m=1}^{N_R} \psi_{im} R_m + \kappa_i R_i = z_i + \chi_i, \quad (8)$$

$$i = \overline{1, N_R}.$$

Relative to unknown  $w_0' = w_0 - \alpha, \theta_0' = \theta_0 - \beta, R_1 \dots R_{N_R}$ , the system of equations (5), (8) coincides with the system (5), (6).

Thus, the supports displacement as a rigid-body (within the framework of the assumptions of the model) will not change the reactions of the supports, and, hence, the stress state of the beam.

Therefore, the support reactions and moments in the beam, as well as its position can be regulated by displacement of only  $N_R - 2$  supports – without changing the starting position of any two supports.

An analysis of the system of linear algebraic equations (5), (6) also makes it possible to conclude about the

statically and kinematically equivalent states of a beam on rigid supports and a beam on elastic supports.

Let us consider a beam on rigid supports when  $z_i = z_i^0$ ,  $\kappa_i = 0$ ,  $i = \overline{1, N_R}$ . The reactions of supports for this case are denoted as  $R_i^0 = 0$ ,  $i = \overline{1, N_R}$ .

They can be determined by solving a system of equations:

$$\sum_{i=1}^{N_R} R_i^0 (l - a_i) + M_{\Sigma} = 0; \quad (9)$$

$$\sum_{i=1}^{N_R} R_i^0 + P_{\Sigma} = 0; \quad (10)$$

$$w_0^0 + a_i \theta_0^0 + \sum_{m=1}^{N_R} \psi_{im} R_m^0 = z_i^0 + \chi_i, \quad i = \overline{1, N_R}. \quad (11)$$

Now consider the same beam (with the same loads) on elastic supports. Let us define additional displacements of its supports  $\Delta z_i$ ,  $i = \overline{1, N_R}$ . Then the total rigid displacements of the supports will be equal:

$$z_i^1 = z_i^0 + \Delta z_i, \quad i = \overline{1, N_R}. \quad (12)$$

Having substituted these expressions in equations (5), (6), an equation for determining unknown reactions  $R_i^1$ ,  $i = \overline{1, N_R}$  can be obtained:

$$\sum_{i=1}^{N_R} R_i^1 (l - a_i) + M_{\Sigma} = 0; \quad (13)$$

$$\sum_{i=1}^{N_R} R_i^1 + P_{\Sigma} = 0; \quad (14)$$

$$w_0^1 + a_i \theta_0^1 + \sum_{m=1}^{N_R} \psi_{im} R_m^1 + \kappa_i R_i^1 = z_i^0 + \Delta z_i + \chi_i, \quad i = \overline{1, N_R}. \quad (15)$$

Let us subtract from the equations of system (13)–(15) the equations of the system (9)–(11), then for increments  $\Delta w_0 = w_0^1 - w_0^0$ ,  $\Delta \theta_0 = \theta_0^1 - \theta_0^0$ ,  $\Delta R_i = R_i^1 - R_i^0$  the system of equations can be obtained:

$$\sum_{i=1}^{N_R} \Delta R_i (l - a_i) = 0; \quad (16)$$

$$\sum_{i=1}^{N_R} \Delta R_i = 0; \quad (17)$$

$$\Delta w_0 + a_i \Delta \theta_0 + \sum_{m=1}^{N_R} \psi_{im} \Delta R_m + \kappa_i \Delta R_i = \Delta z_i, \quad i = \overline{1, N_R}. \quad (18)$$

Let us transform the last equation like this:

$$\Delta w_0 + a_i \Delta \theta_0 + \sum_{m=1}^{N_R} \psi_{im} \Delta R_m + \kappa_i (R_i^1 - R_i^0) = \Delta z_i - \kappa_i R_i^0 \quad (19)$$

or:

$$\Delta w_0 + a_i \Delta \theta_0 + \sum_{m=1}^{N_R} \psi_{im} \Delta R_m + \kappa_i \Delta R_i = \Delta z_i - \kappa_i R_i^0. \quad (20)$$

If to put:

$$\Delta z_i = \kappa_i R_i^0, \quad (21)$$

then the system of equations (16)–(18) will have a trivial solution  $\Delta R_i = 0$ ,  $i = \overline{1, N_R}$ , therefore,  $R_i^1 = R_i^0$ ,  $i = \overline{1, N_R}$ .

Thus, a beam on rigid supports with displacement  $z_i^0$ , and reactions of supports  $R_i^0$ , is statically equivalent to a beam on elastic supports with displacement  $z_i^1 = z_i^0 + \kappa_i R_i^0$ .

The vertical positions of the elastic supports after loading and additional adjustment are equal to:

$$w_i^1 = z_i^0 + \Delta z_i - \kappa_i R_i^1, \quad i = \overline{1, N_R}. \quad (22)$$

If to take additional adjustment parameters according to formula (21), then we obtain:

$$w_i^1 = z_i^0. \quad (23)$$

Thus, the vertical position of the supports after loading and additional adjustment (21) will be equal to the initial rigid supports position. Consequently, the considered beams will also be kinematically equivalent.

Let us consider now the problem of ensuring a specified vertical position  $w_i^1$ ,  $i = \overline{1, N_R}$  of the elastic supports. Their initial position will be denoted as  $z_i^0$ ,  $i = \overline{1, N_R}$ .

From equalities (22), the  $N_R$  of additional equations can be obtained:

$$z_i^0 + \Delta z_i - \kappa_i R_i^1 = w_i^1, \quad i = \overline{1, N_R}. \quad (24)$$

Equations (13)–(15), (24) constitute a system  $2N_R + 2$  of equations for determining unknowns  $w_0^1$ ,  $\theta_0^1$ ,  $R_1^1, \dots, R_{N_R}^1$ ,  $\Delta z_1, \dots, \Delta z_{N_R}^1$ . However, this system allows the following simplifications. Let us determine the values  $\kappa_i R_i^1$  from equations (24) and substitute in equation (15), after that we obtain:

$$w_0^1 + a_i \theta_0^1 + \sum_{m=1}^{N_R} \psi_{im} R_m^1 = w_i^1 - \chi_i, \quad i = \overline{1, N_R}. \quad (25)$$

The system of equations (13), (14), (25) corresponds to a beam on rigid supports with preceding displacements  $w_i^1$ . After the support reactions are determined  $R_i^1$ ,  $i = \overline{1, N_R}$ , the desired displacements will be obtained from the equalities (24):

$$\Delta z_i = w_i^1 - z_i^0 + \kappa_i R_i^1, \quad i = \overline{1, N_R}. \quad (26)$$

In particular, to ensure the vertical position of the elastic supports after loading, the desired adjustment parameters will be obtained:

$$\Delta z_i = -z_i^0 + \kappa_i R_i^1, \quad (27)$$

where:

$R_i^1$  – reactions found for a beam which rests on undisturbed rigid supports ( $z_i = 0$ ,  $i = \overline{1, N_R}$ ).

The direct application of ratios (25)–(27) is possible if rigid initial displacements of the supports  $z_i^0$  are known, which can be determined only by unloading them.

Let us determine the expression for the additional adjustments, when only the complete displacements of the supports are known.

As before, denote the initial rigid displacements of the elastic supports as  $z_i^0, i = 1, N_R$ . Then the system of equations for determining the reactions (5), (6) can be written as follows:

$$\sum_{i=1}^{N_R} R_i^0 (l - a_i) + M_{\Sigma} = 0; \quad (28)$$

$$\sum_{i=1}^{N_R} R_i^0 + P_{\Sigma} = 0; \quad (29)$$

$$w_0^0 + a_i \theta_0^0 + \sum_{m=1}^{N_R} \psi_{im} R_m^0 + \kappa_i R_i^0 = z_i^0 + \chi_i, \quad i = \overline{1, N_R}. \quad (30)$$

The total supports displacement, which can be measured, is written as follows:

$$w_i^0 = z_i^0 - \kappa R_i^0, \quad i = \overline{1, N_R}. \quad (31)$$

Given this, the equation (30) can be rewritten as follows:

$$w_0^0 + a_i \theta_0^0 + \sum_{m=1}^{N_R} \psi_{im} R_m^0 = w_i^0 + \chi_i, \quad i = 1, N_R. \quad (32)$$

Having solved the system of equations (28), (29), (32) the reactions  $R_i^0, i = 1, N_R$  can be found. After this, the rigid displacements of the supports are determined from (31):

$$z_i^0 = w_i^0 + \kappa_i R_i^0, \quad i = \overline{1, N_R}. \quad (33)$$

Having substituted the expressions (33) into formula (26), the expressions for the desired adjustment are obtained:

$$\Delta z_i = w_i^1 - w_i^0 + \kappa_i (R_i^1 - R_i^0). \quad (34)$$

Therefore, the algorithm for determining the necessary adjustment parameters is as follows:

1) measure the initial total displacement of the supports  $w_i^0, i = 1, N_R$ . By solving the system of equations (28), (29), (32), the reactions  $R_i^0, i = 1, N_R$  are determined;

2) specify the desired new displacements of supports  $w_i^1, i = 1, N_R$ . By solving a system of equations similar to (28), (29), (32) (with the superscript "1"), the reactions  $R_i^1, i = 1, N_R$  are determined;

3) calculate the desired adjustment parameters by the formula (34).

Let us now solve the problem of the beam position after the adjustment parameters  $\Delta z_i^0, i = 1, N_R$  are specified, in case when the complete initial positions of the supports  $w_i^0, i = 1, N_R$  are known. The initial reactions of the supports  $R_i^0$  are determined from the system of equations (28), (29), (32).

After adjustment, the reactions of the supports will be equal to  $R_i^1, i = 1, N_R$ . They are determined from the system of equations:

$$\sum_{i=1}^{N_R} R_i^1 (l - a_i) + M_{\Sigma} = 0; \quad (35)$$

$$\sum_{i=1}^{N_R} R_i^1 + P_{\Sigma} = 0; \quad (36)$$

$$w_0^1 + a_i \theta_0^1 + \sum_{m=1}^{N_R} \psi_{im} R_m^1 + \kappa_i R_i^1 = z_i^0 + \Delta z_i + \chi_i. \quad (37)$$

Given that the complete initial displacements of the supports are the following:

$$w_i^0 = z_i^0 - \kappa R_i^0, \quad (38)$$

equation (37) can be rewritten as:

$$w_0^1 + a_i \theta_0^1 + \sum_{m=1}^{N_R} \psi_{im} R_m^1 + \kappa_i R_i^1 = w_i^0 + \kappa_i R_i^0 + \Delta z_i + \chi_i. \quad (39)$$

By solving the system of equations (35), (36), (39), the reactions  $R_i^1, i = 1, N_R$  are determined.

After that, the total displacements of the supports after adjustment can be calculated:

$$w_i^1 = z_i^0 + \Delta z_i - \kappa_i R_i^1 = w_i^0 + \Delta z_i - \kappa_i (R_i^1 - R_i^0). \quad (40)$$

Although equality (40) corresponds to equality (34), the values  $w_i^1$  and  $R_i^1$ , included into it, have a different meaning.

*Practical implications.* The obtained above ratios and the above algorithm for calculating the beam structure of a rotating unit were implemented in a Delphi environment. In order to verify the correctness of the problem solution, there was used the case known in the literature to study the strength of a rotating furnace body set on eight supports (Kuzo, Dziubyk, & Yefremov, 2009). To determine the value of the supports yielding, the range of maximum values ( $\kappa_{\max} = 1,14 \cdot 10^{-3}$  m/MN) and minimum values ( $\kappa_{\min} = 0,81 \cdot 10^{-3}$  m/MN) can be distinguished, obtained in the work (Kuzo & Dziubyk, 2008).

The obtained results of the value of elastic deformations of the supporting nodes, for the case of the initial rectilinear position of the supports are represented in Table 1.

**Table 1. Elastic deformations of supporting nodes**

Support and deformation	1	2	3	4	5	6	7	8
$\delta (\kappa_{\max}),$ mm	3.28	4.61	4.34	4.54	4.68	4.64	5.72	4.37
$\delta (\kappa_{\min}),$ mm	2.32	3.29	3.05	3.22	3.32	3.24	4.11	3.07

The performed calculations have revealed that for the case of accepted values of yielding, a misalignment of the axis of rotation of the unit body on the supports is observed in the range from 2 to 6 mm. Therefore, to reduce the adjusting displacements, a parallel displacement of the axis of body rotation downward relative to the initial position of the supports should be performed. In particular, in the studied case, the displacement values will be 4.51 or 3.20 mm for maximum and minimum yielding, respectively.

Using the obtained system of algebraic equations (5), (6) and the presented algorithm, the calculations have been performed of the adjusting displacements for the case of displacement of the supports taking into account their yielding.

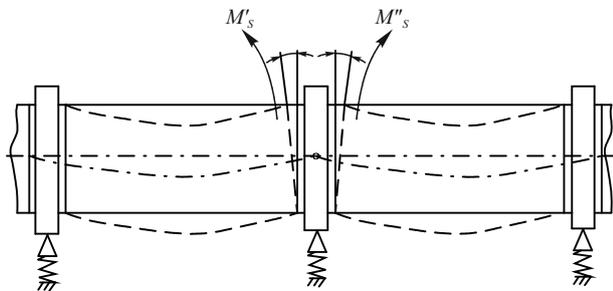
The obtained results of the value of the necessary adjusting displacements are presented in Table 2. In this case, they will not exceed 1.23 mm. Comparison of the obtained results for the case of initially undisplaced supports with the data given in the literature (Kuzo, Dziubyk, & Yefremov, 2009) has testified their complete matching.

**Table 2. Results of adjusting the supporting nodes displacement**

Adjustment of displacement	Support							
	1	2	3	4	5	6	7	8
$\Delta z_i (\kappa_{\max}), \text{ mm}$	1.23	-0.10	0.17	-0.03	-0.17	-0.13	-1.21	0.14
$\Delta z_i (\kappa_{\min}), \text{ mm}$	0.88	-0.09	0.15	-0.02	-0.12	-0.04	-0.91	0.13

The obtained data analysis shows that the presence of elastic deformations in the supports predetermines their displacement by values close to the standard boundaries (Dziubyk, Kuzo, & Prokopyshyn, 2009). In such a case, the presence of previous displacements predetermines a significant redistribution of the acting forces in the rotating furnace body and is determined by their direction. Therefore, it is important to correctly determine the value of adjusting displacements.

Another aspect of using the proposed methodology is to take into account the elastic deformations of the supporting nodes when performing repairs to replace part of the body. Here, as a result of cutting out the defective area, the end crosscuts of the shell ring under the action of bending moments'  $M'_s = M''_s = M_s$  are rotated by a certain angle  $\theta$  (Fig. 2).



**Figure 2. Position of the body when cutting out a defective area**

After completing welding, in this position, various stresses will occur in the end crosscuts during subsequent operation. To avoid this, it is expedient to lower the support before cutting out the defective area of the body, taking into account its elastic deformation, by an appropriate value ( $z_{si}$ ), at which the bending moment  $M_s$  is reduced to zero ( $M_s = 0$ ), and the end crosscuts of the body become parallel.

The proposed methodology for performing repair work enables to create the same working conditions for the body material in specific transverse crosscuts (equal stresses at all points of the annular crosscut during rotation of the unit).

Accounting for elastic deformations of supports was carried out based on the application of the above developed mathematical model. The displacements of the

support as a whole from a specified position is presented in Table 3. The end supports are not considered due to the presence of cantilever elements.

**Table 3. Calculated displacement values ( $z_s$ ) of supports when repairing**

Support displacement ( $z_{si}$ ) at different yielding, mm	Number of support					
	2	3	4	5	6	7
$\kappa_i = 0, i = \overline{1, N_R}$	4.83	2.28	1.91	3.17	2.02	3.92
$z_{si} (\kappa_{\max}), \text{ mm}$	6.05	4.48	3.42	4.98	4.87	4.32
$Z_{si} (\kappa_{\min}), \text{ mm}$	5.86	3.85	3.01	4.47	4.11	4.24

As can be seen, the lack of consideration of the elastic deformations of the supporting nodes elements leads to insufficiently complete setting of parallelism of the furnace body ends. Given the significant overall dimensions of the body, this leads to welding materials overconsumption and a decrease in the efficiency of the repair process. Moreover, the subsequent operation, due to local stress state disturbances, will significantly reduce the residual operation life of the unit.

#### 4. CONCLUSIONS

Based on the problem solution of the static equilibrium of the beam structure, which has piecewise constant flexural rigidity and is placed on elastic regulated supports, a system of linear algebraic equations has been obtained for its calculation. An algorithm is proposed for determining vertical adjusting displacements, taking into account the yielding of the supports. The calculation of the eight-support rotating unit for processing minerals has been performed and the necessary displacements are proposed to ensure the rectilinearity of the axis of rotation.

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#### REFERENCES

- Andreikiv, O.E., Dolins'ka, I.Y., Lysyk, A.R., & Sas, N.B. (2017). Computational model of the propagation of stress-corrosion cracks at high temperatures. *Materials Science*, 52(5), 714-721. <https://doi.org/10.1007/s11003-017-0014-x>
- Andreikiv, O.E., Skal's'kyi, V.R., Dolins'ka, I.Y., & Dzyubyk, A.R. (2018). Influence of corrosive hydrogenating media on the residual service life of structural elements in the maneuvering mode of operation. *Materials Science*, 54(1), 61-68. <https://doi.org/10.1007/s11003-018-0158-3>

- Bilobran, B.S., & Kinash, O.B. (2002). Elastoplastic state of a pipe with nonuniform thickness of the walls under combined loading. *Strength of Materials*, 34(2), 187-195. <https://doi.org/10.1023/a:1015370728505>
- Dreus, A., Kozhevnikov, A., Lysenko, K., & Sudakov, A. (2016). Investigation of heating of the drilling bits and definition of the energy efficient drilling modes. *Eastern-European Journal of Enterprise Technologies*, 3(7(81)), 41-46. <https://doi.org/10.15587/1729-4061.2016.71995>
- Dreus, A.J., Sudakov, A.K., Kozhevnikov, A.A., & Vahalin, J.M. (2016). Study on thermal strength reduction of rock formation in the diamond core drilling process using pulse flushing mode. *Naukovi Visnyk Natsionalnoho Hirnychoho Universytetu*, (3), 5-9.
- Dziubyk, L.V., Kuzo, I.V., & Prokopyshyn, I.A. (2009). Statychna rivnovaha balky zminnoi zhorstkosti na pruzhnykh oporakh z poperednim zmishchenniam. *Mashynoznavstvo*, 11(149), 27-30.
- Dzyubyk, A.R., Nykolyshyn, T.M., & Porokhovs'kyi, Y.V. (2016). Influence of residual stresses on the limit equilibrium of a pipeline with internal crack of arbitrary configuration. *Materials Science*, 52(1), 89-98. <https://doi.org/10.1007/s11003-016-9930-4>
- Gashchuk, P.M., & Zorii, I.L. (2002). Fundamentals of simulation of the dynamics of one-dimensional elastic-rigid mechanical systems. *Journal of Mathematical Sciences*, 109(1), 1303-1311. <https://doi.org/10.1023/a:1013717316185>
- Hashchuk, P., & Zorii, L. (1999). *Liniini modeli dyskretno-neperevnykh mekhanichnykh system*. Lviv, Ukraina: Ukrainski tekhnolohii.
- Jaroszewicz, J., & Zorij, L. (1999). Effect of nonhomogenous material properties on transverse vibrations of elastic cantilevers. *International Applied Mechanics*, 35(6), 633-640. <https://doi.org/10.1007/bf02682189>
- Khomenko, O.Y., Kononenko, M.M., Myronova, I.G., & Sudakov, A.K. (2018). Increasing ecological safety during underground mining of iron-ore deposits. *Naukovi Visnyk Natsionalnoho Hirnychoho Universytetu*, (2), 29-38. <https://doi.org/10.29202/nvngu/2018-2/3>
- Kononenko, M., Khomenko, O., Sudakov, A., Drobot, S., & Lkhagva, T. (2016). Numerical modelling of massif zonal structuring around underground working. *Mining of Mineral Deposits*, 10(3), 101-106. <https://doi.org/10.15407/mining10.03.101>
- Kovalev, A.M., Martynyuk, A.A., Boichuk, O.A., Mazko, A.G., Petryshyn, R.I., Slyusarchuk, V.Y., & Slyn'ko, V.I. (2009). Novel qualitative methods of nonlinear mechanics and their application to the analysis of multifrequency oscillations, stability, and control problems. *Nonlinear Dynamics and Systems Theory*, 9(2), 117-145.
- Kuzo, I., Dziubyk, L., & Yefremov, I. (2009) Rozrakhunok pruzhnykh deformatsii opor ta tochnist diahnostuvannia obertovykh pechei. *Haluzeve Mashynoznavstvo, Budivnytstvo*, 3(25), 135-138.
- Kuzo, I.V., & Dziubyk, L.V. (2007). Vplyv polozhennia heometrychnoi osi na mitsnist obertovykh ahrehativ. *Visnyk NU "Lvivska Politekhnikha". Dynamika, Mitsnist ta Proektuvannia Mashyn i Pryladiv*, (588), 53-57.
- Kuzo, I.V., & Dziubyk, L.V. (2008). Doslidzhennia pruzhnykh deformatsii opornykh vuzliv ta yikh vplyv na sylovi kharakterystyky obertovykh pechei. *Visnyk NU "Lvivska Politekhnikha"*. (613), 106-110.
- Kuzo, I.V., & Dziubyk, L.V. (2009). Vrakhuvannia pruzhnykh deformatsii opor obertovykh pechei pid chas montazhno-nalahodzhuvannykh robot. *Visnyk NU "Lvivska Politekhnikha"*. *Dynamika, Mitsnist ta Proektuvannia Mashyn i Pryladiv*, (641), 39-43.
- Kuzo, I.V., Mykolskiy, Yu.N., & Shevchenko, T.G. (1982). *Sovremennye metody kontrolya oborudovaniya*. Lvov, Ukraina: Vyscha shkola.
- Maksymovych, O.V., Lavrenchuk, S.V., & Solyar, T.Y. (2019). Contact problem for an anisotropic half plane with cracks. *Journal of Mathematical Sciences*, 240(2), 173-183. <https://doi.org/10.1007/s10958-019-04345-3>
- Polivanov, A.A., Belov, A.V., & Morozova, E.V. (2017). Evaluation of stress-strain state of engineering structures subject to damage in materials under creep-based simulation. *Procedia Engineering*, (206), 1464-1469. <https://doi.org/10.1016/j.proeng.2017.10.662>
- Timoshenko, S., & Woinowsky-Krieger, S. (1959). *Theory of plates and shells*. New York, United States: McGraw-Hill.
- Trach, V.M. (2007). Stability of conical shells made of composites with one plane of elastic symmetry. *International Applied Mechanics*, 43(6), 662-669. <https://doi.org/10.1007/s10778-007-0065-z>
- Westergaard, H.M. (2014). *Theory of elasticity and plasticity*. Cambridge, United States: Harvard University Press.
- Zorij, L.M. (1979). O novom metode postroeniya resheniy lineynykh differentsialnykh uravneniy. *Doklady AN USSR*, A(5), 351-355.

## ЗАБЕЗПЕЧЕННЯ ЗАДАНОГО ПОЛОЖЕННЯ БАГАТООПОРНИХ ОБЕРТОВИХ АГРЕГАТИВ ПРИ ЗБАГАЧЕННІ КОРИСНИХ КОПАЛИН

А. Дзюбик, А. Судаков, Л. Дзюбик, Д. Судакова

**Мета.** Розв'язання задачі про встановлення заданого вертикального переміщення опор обертового агрегату із врахуванням податливості в перерізі опорного вузла та особливостей конструктивного виконання.

**Методика.** При виконанні роботи застосовуються підходи, що ґрунтуються на положеннях теорії пружності із врахуванням змінного поперечного перерізу агрегату та різного висотного положення опор. Для встановлення загальних виразів, які описують положення осі досліджуваної багатоопорної конструкції, застосовувався метод функцій Коші.

**Результати.** Отримано систему розв'язуваних алгебраїчних рівнянь, яка дає змогу виконувати обчислення для балкової циліндричної конструкції на жорстких та пружних опорах. Представлено вирази для встановлення величини вертикального зміщення опор із врахуванням експлуатаційних характеристик агрегату, зокрема наявності впливу сусідніх опор та величини діючих навантажень. Розглянуто випадки реалізації регульованих переміщень із застосуванням отриманої системи рівнянь при повному розвантаженні опорних вузлів, а також при використанні даних технічного діагностування – встановлених значень сумарних переміщень опор. Отримано вирази для обчислення шуканих регулювань. Запропоновано алгоритм виконання обчислювальних операцій.

**Наукова новизна.** Отримав подальший розвиток метод регулювання заданого положення опор обертового агрегату, що здійснюється із встановленням пружних складових, жорстких зміщень, повного переміщення в опорному вузлі та результатів технічного діагностування.

**Практична значимість.** Проведено обчислення із врахуванням податливості опор у діапазоні максимальних та мінімальних значень. Показано, що величина зміщення окремих опор, яку необхідно враховувати внаслідок їх податливості, може сягати граничних нормативних значень. Наявність попередніх зміщень опор зумовлює значний перерозподіл діючих зусиль в корпусі обертової печі та визначається їх напрямом. Встановлено величину регулювальних переміщень, які необхідно виконати для отримання прямолінійної осі обертання. Показано, що доцільно реалізувати паралельне зміщення проектованої осі обертання для зменшення регулювань в опорних вузлах.

**Ключові слова:** обертові агрегати, балкові конструкції, регулювання опор, пружні деформації опор, багатопорні агрегати, вісь обертання

## ОБЕСПЕЧЕНИЯ ЗАДАННОГО ПОЛОЖЕНИЯ МНОГООПОРНЫХ ВРАЩАЮЩИХСЯ АГРЕГАТОВ ПРИ ОБОГАЩЕНИИ ПОЛЕЗНЫХ ИСКОПАЕМЫХ

А. Дзюбик, А. Судаков, Л. Дзюбик, Д. Судакова

**Цель.** Решение задачи об установлении заданного вертикального перемещения опор вращающегося агрегата с учетом податливости в сечении опорного узла и особенностей конструктивного исполнения.

**Методика.** При выполнении работы применяются подходы, основанные на положениях теории упругости с учетом переменного поперечного сечения агрегата и различного высотного положения опор. Для установления общих выражений, описывающих положение оси исследуемой многоопорной конструкции, применялся метод функций Коши.

**Результаты.** Получена система решаемых уравнений, которая позволяет выполнять вычисления для балочной цилиндрической конструкции на жестких и упругих опорах. Представлены выражения для установления величины вертикального смещения опор с учетом эксплуатационных характеристик агрегата, в частности наличия влияния соседних опор и величины действующих нагрузок. Рассмотрены случаи реализации регулировочных перемещений с применением полученной системы уравнений при полной разгрузке опорных узлов, а также при использовании данных технического диагностирования – установленных значений суммарных перемещений опор. Получены выражения для вычисления искомых регулировок. Предложен алгоритм выполнения вычислительных операций.

**Научная новизна.** Получил дальнейшее развитие метод регулирования заданного положения опор вращающегося агрегата, который осуществляется с установлением упругих составляющих, жестких смещений, полного перемещения в опорном узле и результатов технического диагностирования.

**Практическая значимость.** Проведены вычисления с учетом податливости опор в диапазоне максимальных и минимальных значений. Показано, что величина смещения отдельных опор, которую необходимо учитывать в результате их податливости, может достигать предельных нормативных значений. Наличие предыдущих смещений опор обуславливает значительное перераспределение действующих усилий в корпусе вращающейся печи и определяется их направлением. Установлено величину регулировочных перемещений, которые необходимо выполнить для получения прямолинейной оси вращения. Показано, что целесообразно реализовать параллельное смещение проектируемой оси вращения для уменьшения регулировок в опорных узлах.

**Ключевые слова:** вращающиеся агрегаты, балочные конструкции, регулирование опор, упругие деформации опор, многоопорные агрегаты, ось вращения

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