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THE IMPACT OF VIBRATION MECHANISM' INSTALLATION PLACE ON THE PROCESS OF RETRIEVING STUCK DRILL PIPE

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ВПЛИВ МІСЦЯ УСТАНОВКИ ВІБРАЦІЙНОГО МЕХАНІЗМУ НА ПРОЦЕС ВИВІЛЬНЕННЯ ПРИХОПЛЕНОГО БУРИЛЬНОГО ІНСТРУМЕНТУ

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ABSTRACT

Purpose. The present paper studies the dependence of the dynamic oscillatory movements in the area of drill string sticking on the parameters of vibrating mechanism and place of its installation in the heavy weight drill pipe. The authors focus specifically on conditions responsible for resonant vibrations excitation in the drill string, which facilitates the release of stuck drill pipe. The aim is to use the results of the research in the development of the method for vibrator application, to work out recommendations concerning the choice of its installation place in the heavy weight drill pipe, its tuning to the resonant frequency, the choice of induced force amplitude.

Methods. The authors proposed a mathematical model of the drill string functioning with vibration mechanism installed into the heavy weight drill pipe. The vibrator is needed to eliminate sticking of the drilling tool. The suggested discrete-continuum model which takes wave processes into account allows to conduct grounded experimental research.

Findings. The developed mathematical model served as the basis for designing a computer program aimed to visualize oscillatory processes taking place in the string of pipes, and to calculate basic dynamic and kinematic characteristics of the analyzed system. Frequency oscillation spectrum was calculated for the selected layout of the drill string and the resonant amplitudes at these frequencies.

Originality. The proposed technique allows to significantly determine strains, stresses and safety margins in the arbitrary drill string section more accurately and to predict its sticking while drilling oil and gas boreholes.

Practical implications. A specific method of selecting a place for the vibrator installation was elaborated. Recommendations concerning the choice of installation site, amplitude of excitation force, resonant frequencies for elimination of pipes' sticking and prevention of drill strings destruction are provided.

Keywords: *drilling, drill string, elastic waves, mathematical model, vibrations, sticking, vibrator, stress, energy of the stuck drill string*

1. INTRODUCTION

Drilling engineers pay great attention to the trouble-free drilling of oil and gas wells. It is explained by costly and long-term maintenance works aimed to liquidate breakdowns especially those caused by sticking of drilling tool, which occur far more often than other accidents (Wen, Chen, & Dong, 2014).

Stuck-pipe problems are still considered (Bodine, 1994) as potentially dangerous, despite a great number of publications, developed modern drilling technologies and methods for prevention of drill string (DS) sticking. Thereby, one cannot underestimate the importance of elaborating new technological solutions and appropriate techniques for the stuck DS retrieval (Houlbrook & Lyon, 2006).

Current methods of solving stuck-pipe problems are based on the use of physical, mechanical and hydraulic impact on the sticking zone (Liu, Wang, Li, & Xu, 2011). It leads to weakening of wall rock pressure forces because of rock dissolution, and, consequently, reduction of friction. Hydraulic methods are based on the alteration of

hydraulic pressure in the accident zone by way of regulating hydro-statistic pressure component, or forming hydraulic waves of the drilling mud. Mechanical methods are based on creation of vibration disturbance, a shock or explosive loading, applied in the zone of DS sticking (Abraham & Marsden, 2008).

A considerable amount of experimental researches and engineering tests have proved that the mechanical methods are especially effective. One of the most advanced methods of retrieving stuck DS is the use of vibration mechanisms, given simplicity of their design. These devices allow to realize a big variety of operational modes, beginning from excitation of disturbing forces to selection of vibration mechanism' parameters (Bailey & Gupta, 2008). Moreover, vibrators are very durable.

2. MATHEMATICAL MODEL OF ELASTIC WAVES EMITTED BY A DRILL PIPE WITH A VIBRATOR

Mechanical scheme of DS with a vibrating mechanism can be represented by a discrete-continuous system comprising six sections (Fig. 1).

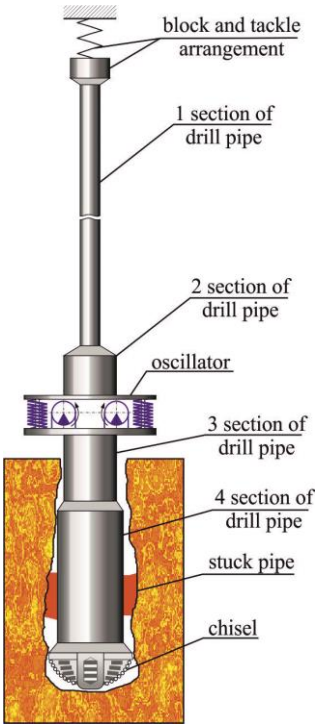

Figure 1. Drill string with built-in vibrator

It will examine the dynamic characteristics of the DS vibrator, built in heavy weight drill pipe (HWDP). The first section is composed of cylindrical steel pipes made of steel with density ρ_1 = 7869,5 kg/m^3, outside diameter D_1 = 127 mm, inside diameter d_1 = 107 mm and the total length l_1 = 1400 m. The second section is a part of HWDP located above the vibrator, the third section is located below the vibrator. The stuck part of the DS is also divided into three sections: the fourth – loose area above the sticking zone, fifth – the stuck part of DS, sixth – loose area located below the sticking zone.

The length of the heavy weight drill pipe is $l_2 + l_3$ = 130 m, and the length of the stuck drill pipe – $l_4 + l_5 + l_6$ = 40 m. These pipes are made of steel with density ρ_1 = 7759 kg/m^3, outside diameter D_i = 177,8 mm and inside diameter d_1 = 71,4 mm, i = 2 – 6. The vibrator installed in the heavy weight drill pipe. Its location is determined by the length of the second section $0 < l_2 < 130$ m.

The mathematical model for the proposed scheme of the drill string describes dynamic processes in DS. According to the theory of elasticity (Giovine, 2012), wave differential equations for the cross-cuts of each section can be written as:

$$\begin{aligned}
&\ddot{u}_1(x_1,t)+2h_1\dot{u}_1(x_1,t)-a_1^2u_1''(x_1,t)=g;\\
&\ddot{u}_2(x_2,t)+2h_2\dot{u}_2(x_2,t)-a_2^2u_2''(x_2,t)=g;\\
&\ddot{u}_3(x_3,t)+2h_3\dot{u}_3(x_3,t)-a_2^2u_3''(x_3,t)=g;\\
&\ddot{u}_4(x_4,t)+2h_4\dot{u}_4(x_4,t)-a_3^2u_4''(x_4,t)=g;\\
&\ddot{u}_5(x_5,t)+2h_5\dot{u}_5(x_5,t)-a_3^2u_5''(x_5,t)=\\
&=g-f_0 sign\left[\dot{u}_5(x_5,t)\right];\\
&\ddot{u}_6(x_6,t)+2h_6\dot{u}_6(x_6,t)-a_3^2u_6''(x_6,t)=g,
\end{aligned} \tag{1}$$

where:

$$\ddot{u}_i(x_i,t)=\frac{\partial^2 u_i(x_i,t)}{\partial t^2};$$

$h_i=\dfrac{\alpha_i}{2m_i}$, $i=1-6$ – multiple coefficient of viscous resistance;

$\alpha_1=48.4$ kg/c, $\alpha_i=380.4$ kg/c, $i=2-6$ – coefficients drilling fluid interaction with DS sections;

$m_1=\rho_1F_1l_1$ – mass of the first section;

$m_i=\rho_2F_2l_i$, $i=2,3$ – mass of the second and the third sections of HWDP accordingly;

$m_i=\rho_3F_3l_i$, $i=4-6$ – mass of the fourth, fifth and sixth sections of the stuck drill pipe accordingly;

F_i, $i=1-3$ – cross sectional areas of drill pipes;

$a_i=\sqrt{E_i/\rho_i}$, $i=1-3$ – speed of elastic waves propagation;

$E_1=E_2=E_3=210$ GPa – modulus of elasticity or Young's modulus;

$u_i''(x_i,t)=\dfrac{\partial^2 u_i(x_i,t)}{\partial x_i^2}$, $i=1-6$;

g – gravitation acceleration;

$f_0=0.3$ – coefficient of DS dry rod friction;

x_i, $i=1-6$ – current coordinate of DS i-th section cross-cut;

t – current time.

Moving parts of the hoisting system are presented by multiple mass $m_0=9855$ kg and rigidity of drilling lines $c_0=53$ MN/m. We assume that the vibrator produces harmonic disturbing force $P(t)$.

Next, we have to attach boundary conditions at the ends and docking sections of DS to the dynamic equations of the drill string motion (Han, Kim & Karkoub, 2014):

– at the top of DS is hoisting system- the first section of the drill string:

$$E_1F_1u_1'(0,t)+m_0g=c_0u_1(0,t)+m_0a_1^2u_1''(0,t); \tag{2}$$

– at the docking of the 1st and 2nd sections of drill pipes:

$$u_1(l_1,t)=u_2(0,t),$$
$$E_1F_1u_1'(l_1,t)=E_2F_2u_2'(0,t)+q(F_2-F_1)gl_1; \quad (3)$$

– at the docking of the second and third sections of drill pipes:

$$u_2(l_2,t)=u_3(0,t),$$
$$E_2F_2\left[u_3'(0,t)-u_2'(l_2,t)\right]=-P(t); \quad (4)$$

– at the docking of the third and fourth sections of drill pipes:

$$u_3(l_3,t)=u_4(0,t),$$
$$E_2F_2u_3'(l_3,t)=E_3F_3u_4'(0,t)+q(F_3-F_2)g\sum_{j=1}^{3}l_j; \quad (5)$$

– at the docking of the fourth-fifth and fifth-sixth sections of drill pipe:

$$u_4(l_4,t)=u_5(0,t), \qquad u_4'(l_4,t)=u_5'(0,t),$$
$$u_5(l_5,t)=u_6(0,t), \qquad u_5'(l_5,t)=u_6'(0,t); \quad (6)$$

– at the lower end of the drill string:

$$E_3u_6'(l_6,t)=-qg\sum_{j=1}^{6}l_j \quad \text{or} \quad E_3F_3u_6'(l_6,t)=R_r, \quad (7)$$

where:

q = 1200 kg/m^3 – the density of drilling fluid;

R_r – rock reaction force on the bore bit.

From static equilibrium equations of the mechanical system we got the initial motion conditions $u_i(x_i, 0)$, i = 1 – 6 for the system of equations (1):

– for the current cross-cut of the first DS section ($0\le x_1\le l_1$):

$$u_1(x_1,0)=\frac{m_0}{c_0}g+\left(a_{1S}-\frac{P_0}{g}l_5\right)\left(\frac{1}{c_0}+\frac{x_1}{E_1F_1}\right)g-\frac{\rho_1 g}{2E_1}x_1^2; \quad (8)$$

– for the current cross-cut of the second section DS ($0\le x_2\le l_2$, l_2 – place of the vibrating mechanism installation):

$$u_2(x_2,0)=\frac{m_0}{c_0}g+\left(a_{1S}-\frac{P_0}{g}l_5\right)\left(\frac{1}{c_0}+\frac{l_1}{E_1F_1}\right)g+\left(a_{2S}-\frac{P_0}{g}l_5\right)\frac{gx_2}{E_2F_2}-\frac{g}{2}\sum_{j=1}^{2}\frac{\rho_j}{E_j}x_j^2; \quad (9)$$

– for the current cross-cut of the third section DS ($0\le x_3\le l_3$):

$$u_3(x_3,0)=\left(m_0+a_{1S}-\frac{P_0}{g}l_5\right)\frac{g}{c_0}+g\sum_{j=1}^{2}\left(a_{jS}-\frac{P_0}{g}l_5-\frac{m_j}{2}\right)\frac{l_j}{E_jF_j}+\left(a_{3S}-\frac{P_0}{g}l_5\right)\frac{gx_3}{E_2F_2}-\frac{g\rho_2}{2E_2}x_3^2; \quad (10)$$

– for the current cross-cut of the fourth section – loose part of the stuck DS, located above the sticking zone, ($0\le x_4\le l_4$):

$$u_4(x_4,0)=\left(m_0+a_{1S}-\frac{P_0}{g}l_5\right)\frac{g}{c_0}+g\sum_{j=1}^{2}\left(a_{jS}-\frac{P_0}{g}l_5-\frac{m_j}{2}\right)\frac{l_j}{E_jF_j}+\left(a_{3S}-\frac{P_0}{g}l_5-\frac{m_3}{2}\right)\frac{gl_3}{E_2F_2}+$$
$$+\left(a_{4S}-\frac{P_0}{g}l_5\right)\frac{gx_4}{E_3F_3}-\frac{g\rho_3}{2E_3}x_4^2; \quad (11)$$

– for the current cross-cut of the stuck DS part ($0\le x_5\le l_5$, l_5 – length of sticking):

$$u_5(x_5,0)=\left(m_0+a_{1S}-\frac{P_0}{g}l_5\right)\frac{g}{c_0}+\left(a_{1S}-\frac{P_0}{g}l_5-\frac{m_1}{2}\right)\frac{gl_1}{E_1F_1}+g\sum_{j=2}^{3}\left(a_{jS}-\frac{P_0}{g}l_5-\frac{m_j}{2}\right)\frac{l_j}{E_2F_2}+$$
$$+\left(a_{4S}-\frac{P_0}{g}l_5-\frac{m_4}{2}\right)\frac{gl_4}{E_3F_3}+\left(a_{5S}-\frac{P_0}{g}l_5\right)\frac{gx_5}{E_3F_3}-\frac{(g\rho_3F_3-P_0)}{2E_3F_3}x_5^2; \quad (12)$$

– for the current cross-cut of the stuck DS loose part, located below the sticking zone ($0\le x_6\le l_6$):

$$u_6(x_6,0)=\left(m_0+a_{1S}-\frac{P_0}{g}l_5\right)\frac{g}{c_0}+g\left(a_{1S}-\frac{P_0}{g}l_5-\frac{m_1}{2}\right)\frac{l_1}{E_1F_1}+g\sum_{j=2}^{3}\left(a_{jS}-\frac{P_0}{g}l_5-\frac{m_j}{2}\right)\frac{l_j}{E_2F_2}+$$
$$+\sum_{j=4}^{5}\left(a_{jS}-\frac{P_0}{g}l_5-\frac{m_j}{2}\right)\frac{l_j}{E_3F_3}+\left[\rho_3\left(l_6-\frac{1}{2}x_6\right)-q\sum_{j=1}^{6}l_j\right]\frac{gx_6}{E_3}, \quad (13)$$

where:

P_0 – evenly distributed load intensity of DS stuck part;

$$a_{1S}=(\rho_1-q)F_1l_1+(\rho_2-q)F_2\sum_{j=2}^{3}l_j+(\rho_3-q)F_3\sum_{j=4}^{6}l_j;$$

$$a_{2S}=\left(\rho_2\sum_{j=2}^{3}l_j-q\sum_{j=1}^{3}l_j\right)F_2+(\rho_3-q)F_3\sum_{j=4}^{6}l_j;$$

$$a_{3S}=\left(\rho_2l_3-q\sum_{j=1}^{3}l_j\right)F_2+(\rho_3-q)F_3\sum_{j=4}^{6}l_j;$$

$$a_{4S}=\left(\rho_3\sum_{j=4}^{6}l_j-q\sum_{j=1}^{6}l_j\right)F_3;$$

$$a_{5S}=\left(\rho_3\sum_{j=5}^{6}l_j-q\sum_{j=1}^{6}l_j\right)F_3.F.$$

At the beginning of the motion in the position of static equilibrium speeds in the current DS cross-cuts are:

$$\dot{u}_i(x_i,0)=0,\ i=1-6. \quad (14)$$

Since at the initial time ($t=0$) all cross-cuts of the DS stuck part are motionless $u_5(x_5,0)=0$, from equation (12) we determine the place l_4 and length l_5 of the sticking, and intensity of the load P_0.

3. DYNAMICS OF CROSS SECTION DRILLING STRING

Since the differential equations (1) are constantly inhomogeneous equations of the second kind (Kelly, 2008), the stationary solutions have the form of a quadratic expression. Static deformations of the current DS cross-cuts, satisfying the system of equations (1) and boundary conditions (2) – (7) are of the form:

$$
\begin{aligned}
&u_1(x_1)=\frac{m_0}{c_0}g+(a_{1S}-\mu m_5)\left(\frac{1}{c_0}+\frac{x_1}{E_1F_1}\right)g-\frac{g\rho_1}{2E_1}x_1^2;\\
&u_2(x_2)=\frac{m_0}{c_0}g-\frac{\rho_1}{2E_1}gl_1^2+(a_{1S}-\mu m_5)\left(\frac{1}{c_0}+\frac{l_1}{E_1F_1}\right)g+(a_{2S}-\mu m_5)\frac{gx_2}{E_2F_2}-\frac{g\rho_2}{2E_2}x_2^2;\\
&u_3(x_3)=(m_0+a_{1S}-\mu m_5)\frac{g}{c_0}+g\sum_{j=1}^{2}\left(a_{jS}-\mu m_5-\frac{m_j}{2}\right)\frac{l_j}{E_jF_j}+(a_{3S}-\mu m_5)\frac{gx_3}{E_2F_2}-\frac{g\rho_2}{2E_2}x_3^2;\\
&u_4(x_4)=(m_0+a_{1S}-\mu m_5)\frac{g}{c_0}+g\sum_{j=1}^{2}\left(a_{jS}-\mu m_5-\frac{m_j}{2}\right)\frac{l_j}{E_jF_j}+\left(a_{3S}-\mu m_5-\frac{m_3}{2}\right)\frac{gl_3}{E_2F_2}+\\
&+(a_{4S}-\mu m_5)\frac{gx_4}{E_3F_3}-\frac{g\rho_3}{2E_3}x_4^2;\\
&u_5(x_5)=(m_0+a_{1S}-\mu m_5)\frac{g}{c_0}+\left(a_{1S}-\mu m_5-\frac{m_1}{2}\right)\frac{gl_1}{E_1F_1}+g\sum_{j=2}^{3}\left(a_{jS}-\mu m_5-\frac{m_j}{2}\right)\frac{l_j}{E_2F_2}+\\
&+\left(a_{4S}-\mu m_5-\frac{m_4}{2}\right)\frac{gl_4}{E_3F_3}+(a_{5S}-\mu m_5)\frac{g\rho_3}{E_3}l_5x_5-\frac{g\rho_3}{2E_3}(1-\mu)x_5^2;\\
&u_6(x_6)=(m_0+a_{1S}-\mu m_5)\frac{g}{c_0}+g\left(a_{1S}-\mu m_5-\frac{m_1}{2}\right)\frac{l_1}{E_1F_1}+g\sum_{j=2}^{3}\left(a_{jS}-\mu m_5-\frac{m_j}{2}\right)\frac{l_j}{E_2F_2}+\\
&+g\sum_{j=4}^{5}\left(a_{jS}-\mu m_5-\frac{m_j}{2}\right)\frac{l_j}{E_3F_3}+\left[\rho_3\left(l_6-\frac{1}{2}x_6\right)-q\sum_{j=1}^{6}l_j\right]\frac{gx_6}{E_3},
\end{aligned}
\tag{15}
$$

where:

$$\mu=(f_0/g)sign[\dot{u}_5(x_5,t)].$$

Stationary solutions (15) testify that elastic motions of DS cross-cuts depend on the physical characteristics of the hoisting system and drill pipes, forces of dry friction (μ) and the density of fluids patch (q). They define the location, length and integrated power of rock pressure on DS. If the elastic deformation in some cross-cut starts to decrease, it shows that DS mobility is limited and there is propability of sticking occurence. By changing the density of drilling fluid we can reduce the sticking force and the intensity of rock pressure or the sticking length. Equations (15) completely determine the efficiency of breakdown elimination by this method. However, if implementation of this technique in practice fails, it becomes necessary to use mechanical methods (Velten, 2009).

Given the stationary solutions (15) a system of homogeneous differential equations was obtained:

$$
\begin{aligned}
&\ddot{u}_{1d}(x_1,t)+2h_1\dot{u}_{1d}(x_1,t)-a_1^2u''_{1d}(x_1,t)=0;\\
&\ddot{u}_{id}(x_i,t)+2h_i\dot{u}_{id}(x_i,t)-a_2^2u''_{id}(x_i,t)=0,\ i=2,3;\\
&\ddot{u}_{id}(x_i,t)+2h_i\dot{u}_{id}(x_i,t)-a_3^2u''_{id}(x_i,t)=0,\ i=4-6
\end{aligned}
\tag{16}
$$

with the inhomogeneous boundary conditions:

$$
\begin{aligned}
&E_1F_1u'_{1d}(0,t)=c_0u_{1d}(0,t)+m_0a_1^2u''_{1d}(0,t);\\
&E_1F_1u'_{1d}(l_1,t)=E_2F_2u'_{2d}(0,t);\\
&E_2F_2\left[u'_{3d}(0,t)-u'_{2d}(l_2,t)\right]=-P(t);\\
&E_2F_2u'_{3d}(l_3,t)=E_3F_3u'_{4d}(0,t);\\
&u_{id}(l_i,t)=u_{(i+1)d}(0,t),\quad i=1-5;\\
&u'_{id}(l_i,t)=u'_{(i+1)d}(0,t),\ i=4,5;\\
&u'_{6d}(l_6,t)=0
\end{aligned}
\tag{17}
$$

and initial conditions that satisfy the following equalities:

$$
\begin{aligned}
&u_{id}(x_i,0)=u_i(x_i,0)-u_i(x_i);\\
&\dot{u}_{id}(x_i,0)=\dot{u}_i(x_i,0),\quad i=1-6
\end{aligned}
\tag{18}
$$

and according to the conditions obtained above (8) – (15) take the form:

$$
\begin{aligned}
&u_{1d}(x_1,0)=\left(\frac{1}{c_0}+\frac{x_1}{E_1F_1}\right)a_0;\\
&u_{2d}(x_1,0)=\left(\frac{1}{c_0}+\frac{l_1}{E_1F_1}+\frac{x_2}{E_2F_2}\right)a_0;\\
&u_{3d}(x_3,0)=\left(\frac{1}{c_0}+\sum_{j=1}^{2}\frac{l_j}{E_jF_j}+\frac{x_3}{E_3F_3}\right)a_0;\\
&u_{4d}(x_4,0)=\left(\frac{1}{c_0}+\sum_{j=1}^{2}\frac{l_j}{E_jF_j}+\frac{x_3}{E_2F_2}+\frac{x_4}{E_3F_3}\right)a_0;\\
&u_{5d}(x_5,0)=\left(\frac{1}{c_0}+\frac{l_1}{E_1F_1}+\frac{1}{E_2F_2}\sum_{j=2}^{3}l_j+\frac{l_4+x_5}{E_3F_3}-\frac{x_5^2}{2E_3F_3l_5}\right)a_0;\\
&u_{6d}(x_6,0)=\left(\frac{1}{c_0}+\frac{l_1}{E_1F_1}+\frac{1}{E_2F_2}\sum_{j=2}^{3}l_j+\frac{1}{E_3F_3}\sum_{j=4}^{5}l_j\right)a_0;\\
&\dot{u}_{id}(x_i,0)=0,\quad i=\overline{1,6},
\end{aligned}
\tag{19}
$$

where:

$$a_0=\mu m_5g-P_0l_5.$$

The solution of the equation system (16) is complicated by the fact that the boundary conditions (17) are inhomogeneous because they contain a non-stationary component – a disturbing force $P(t)$. First it is necessary to find such special function $\varphi_i(x_i, t)$, $i = 1 – 6$, so as to ensure homogeneous boundary conditions for new unknown functions $w_i(x_i, t)$. After selecting special functions, the sought-for dynamic movements of DS cross-cuts $u_{id}(x_i, t)$ must be shifted to $\varphi_i(x_i, t)$.

$$w_i\left(x_i,t\right)=u_{id}\left(x_i,t\right)-\varphi_i\left(x_i,t\right),\quad i=\overline{1,6}. \tag{20}$$

Functions $\varphi_i(x_i, t)$, describe additional components of DS forced oscillations (Maugin, 2007). They should ensure homogeneous boundary conditions for functions $w_i(x_i, t)$ and are completely defined by inhomogeneous boundary conditions (17):

$$\varphi_1\left(x_1,t\right)=\varphi_2\left(x_2,t\right)=0;$$

$$\varphi_3\left(x_3,t\right)=\frac{P(t)}{E_2F_2l_3}x_3\left(x_3-l_3\right);$$

$$\varphi_i\left(x_i,t\right)=(-1)^i\frac{P(t)}{E_3F_3l_i}x_i\left(l_i-x_i\right);\ i=4,5; \tag{21}$$

$$\varphi_6\left(x_6,t\right)=\frac{P(t)}{E_3F_3l_6}x_6\left(2l_6-x_6\right).$$

The obtained laws of motion indicate that the disturbing force $P(t)$ excites vibrations below the vibrator installation zone. Equations (21) describe forced components of drill pipes cross-cuts oscillations, excited by vibration force $P(t)$. For the purpose of the research we assumed vibration force to be harmonic $P(t) = A\sin(\omega t + \gamma)$.

Thus, the general boundary problem for functions $u_{id}(x_i, t)$ is reduced to the problem with homogeneous boundary conditions:

$$\begin{aligned}
&E_1F_1w_1'(0,t)=c_0w_1(0,t)+m_0a_1^2w_1''(0,t);\\
&E_1F_1w_1'(l_1,t)=E_2F_2w_2'(0,t);\\
&E_2F_2w_3'(l_3,t)=E_3F_3w_4'(0,t);\\
&w_i(l_i,t)=w_{i+1}(0,t),\ i=\overline{1,5};\\
&w_i'(l_i,t)=w_{i+1}'(0,t),\ i=4,5;\ w_6'(l_6,t)=0.
\end{aligned} \tag{22}$$

For functions $w_i(x_i, t)$ which are solutions to inhomogeneous differential equations obtained in (20) and (21).

$$\begin{aligned}
&\ddot{w}_i\left(x_i,t\right)+2h_i\dot{w}_i\left(x_i,t\right)-a_i^2w_i''\left(x_i,t\right)=0;\\
&\ddot{w}_3\left(x_3,t\right)+2h_3\dot{w}_3\left(x_3,t\right)-a_2^2w_3''\left(x_3,t\right)=q_3\left(x_3,t\right);\\
&\ddot{w}_i\left(x_i,t\right)+2h_i\dot{w}_i\left(x_i,t\right)-a_3^2w_i''\left(x_i,t\right)=q_i\left(x_i,t\right),
\end{aligned} \tag{23}$$

where:

$$q_3\left(x_3,t\right)=\frac{A}{E_2F_2l_3}\left[\left(x_3\left(l_3-x_3\right)\omega^2+2a_2^2\right)\sin\left(\omega t+\gamma\right)-2h_3x_3\left(l_3-x_3\right)\omega\cos\left(\omega t+\gamma\right)\right];$$

$$q_i\left(x_i,t\right)=(-1)^i\frac{A}{E_3F_3l_i}\left[\left(x_i\left(x_i-l_i\right)\omega^2+2a_3^2\right)\sin\left(\omega t+\gamma\right)-2h_ix_i\left(x_i-l_i\right)\omega\cos\left(\omega t+\gamma\right)\right],\ i=4,5;$$

$$q_6\left(x_6,t\right)=\frac{A}{2E_3F_3l_6}\left[\left(x_6\left(x_6-2l_6\right)\omega^2+2a_3^2\right)\sin\left(\omega t+\gamma\right)-2h_6x_6\left(x_6-2l_6\right)\omega\cos\left(\omega t+\gamma\right)\right].$$

Taking into account (19), (20) and (21) initial conditions for the DS current cross-cuts to the sticking zone took the form:

$$\begin{aligned}
&w_1\left(x_1,0\right)=\left(\frac{1}{c_0}+\frac{x_1}{E_1F_1}\right)a_0;\\
&w_2\left(x_2,0\right)=\left(\frac{1}{c_0}+\frac{l_1}{E_1F_1}+\frac{x_2}{E_2F_2}\right)a_0;\\
&w_3\left(x_3,0\right)=\left(\frac{1}{c_0}+\sum_{j=1}^{2}\frac{l_j}{E_jF_j}+\frac{x_3}{E_3F_3}\right)a_0-\frac{A\sin\gamma}{E_2F_2l_3}x_3\left(x_3-l_3\right)\\
&\dot{w}_i\left(x_i,0\right)=0,\ i=1,2;\\
&\dot{w}_3\left(x_3,0\right)=A\omega\frac{l_3-x_3}{E_3F_3l_4}x_3\cos\gamma;
\end{aligned} \tag{24}$$

To find a non-trivial solution to the system of inhomogeneous equations (16) related to system (23) We will use the Fourier's method according to which the DS cross-cuts moving will be expressed by the product:

$$w_i\left(x_i,t\right)=X_i\left(x_i\right)\cdot T_i(t),\ i=1-6, \tag{25}$$

where:

$X_i\left(x_i\right)$ – the function of the current cross-cut;

$T_i(t)$ – the function of the current time.

After substituting relationships (25) into homogeneous dynamic equations which relate to (23), we received a differential equations system of the current cross-cut functions:

$$\begin{aligned}
&X_1''\left(x_1\right)+a_1^2p_k^2X_1\left(x_1\right)=0;\\
&X_i''\left(x_i\right)+a_2^2p_k^2X_i\left(x_i\right)=0,\ i=2,3;\\
&X_i''\left(x_i\right)+a_3^2p_k^2X_i\left(x_i\right)=0,\ i=4,5,6.
\end{aligned} \tag{26}$$

Thus, we have obtained the Sturm-Liouville problem with the following non-trivial solutions to $X_i\left(x_i\right)$:

$$\begin{aligned}
&X_1\left(x_1\right)=A1_1\sin\left(\frac{p_k}{a_1}x_1\right)+A2_1\cos\left(\frac{p_k}{a_1}x_1\right);\\
&X_i\left(x_i\right)=A1_i\sin\left(\frac{p_k}{a_2}x_i\right)+A2_i\cos\left(\frac{p_k}{a_2}x_i\right),\ i=2,3;\\
&X_i\left(x_i\right)=A1_i\sin\left(\frac{p_k}{a_3}x_i\right)+A2_i\cos\left(\frac{p_k}{a_3}x_i\right),\ i=4,5,6.
\end{aligned} \tag{27}$$

After substituting (27) into boundary conditions (22), we obtain a frequency equation for vibrations of the drill string:

$$\begin{aligned}
&\left[\left(\mu_2-\mu_1\operatorname{tg}\frac{p_k}{a_3}l_5\right)\frac{M_3}{\mu_3}-\left(\operatorname{tg}\frac{p_k}{a_3}l_4+\operatorname{tg}\frac{p_k}{a_3}l_5\right)M_4\right]M_1-\left[\left(1+\operatorname{tg}\frac{p_k}{a_3}l_4\operatorname{tg}\frac{p_k}{a_3}l_5\right)\frac{\mu_2}{\mu_3}M_4-\left(\operatorname{tg}\frac{p_k}{a_3}l_4-\right.\right.\\
&\left.\left.-\operatorname{tg}\frac{p_k}{a_3}l_5\right)M_3\right]M_2=\left\{\left[\left(1-\operatorname{tg}\frac{p_k}{a_3}l_4\operatorname{tg}\frac{p_k}{a_3}l_5\right)M_4+\left(\mu_1\operatorname{tg}\frac{p_k}{a_3}l_4+\mu_2\operatorname{tg}\frac{p_k}{a_3}l_5\right)\frac{M_3}{\mu_3}\right]M_1+\left[\left(\operatorname{tg}\frac{p_k}{a_3}l_4\times\right.\right.\right.\\
&\left.\left.\left.\times\operatorname{tg}\frac{p_k}{a_3}l_5+1\right)M_3+\left(\operatorname{tg}\frac{p_k}{a_3}l_4-\operatorname{tg}\frac{p_k}{a_3}l_5\right)\frac{\mu_2}{\mu_3}M_4\right]M_2\right\}\operatorname{tg}\frac{p_k}{a_3}l_6,
\end{aligned} \tag{28}$$

where:

$$M_1 = \left(1 - \frac{\mu_1 p_k}{c_0 - m_0 p_k^2} \operatorname{tg} \frac{p_k}{a_1} l_1\right) \frac{\mu_1}{\mu_2};$$

$$M_2 = \frac{\mu_1 p_k}{c_0 - m_0 p_k^2} + \operatorname{tg} \frac{p_k}{a_1} l_1;$$

$$M_3 = 1 - \operatorname{tg} \frac{p_k}{a_2} l_2 \operatorname{tg} \frac{p_k}{a_2} l_3;$$

$$M_4 = \operatorname{tg} \frac{p_k}{a_2} l_2 + \operatorname{tg} \frac{p_k}{a_2} l_3; \quad \mu_i = F_i \sqrt{E_i \rho_i}, \quad i = 1-3.$$

Frequency equation (28) is transcendent and has an unlimited number of solutions, so the frequency spectrum p_k ($k = 1, \ldots, \infty$) of DS sections natural oscillations is aliquant in character. It should also be noted that the DS frequency value depends on the parameters of all sections of the drill string and the multiple mass and rigidity of the hoisting system. The mathematical model of DS with a vibrating device built in HWDP was compiled to conduct parametric research (Fitzgibbon, Kuznetsov, Neittaanmäki, & Pironneau, 2014). This model allowed to calculate with reasonable accuracy movement and speed of DS cross-cuts, amplitude-frequency, phase-frequency and dynamic characteristics of the drill string, excited by the vibrating mechanism. For the selected DS layout we calculated the frequency spectrum of free oscillations, the first ten of which are presented in Table 1 (Heinz, 2011).

Table 1. Natural frequencies of the drill string

Frequency	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
Hz	3.45	12.70	23.45	34.50	45.50	56.20	65.97	74.30	82.87	91.61

In general, the frequency spectrum of DS natural oscillations is very wide, and the generated wave propagates along the pipe string and is transferred to the place of drill pipes' sticking.

Free oscillations can be amplified by the excitation of forced resonance oscillations by the vibrating device. With an accuracy to an arbitrary multiplier equal to one, natural frequencies p_k are related to their own functions:

$$\begin{aligned}
&X_{1k}(x_1) = \sin \frac{p_k}{a_1} x_1 + \frac{\mu_1 p_k}{c_0 - m_0 p_k^2} \cos \frac{p_k}{a_1} x_1; \quad X_{2k}(x_2) = \left(M_1 \sin \frac{p_k}{a_2} x_2 + M_2 \cos \frac{p_k}{a_2} x_2\right) \cos \frac{p_k}{a_1} l_1; \\
&X_{3k}(x_3) = \left[M_1 \sin \frac{p_k}{a_2}(x_3 + l_2) + M_2 \cos \frac{p_k}{a_2}(x_3 + l_2)\right] \cos \frac{p_k}{a_1} l_1; \\
&X_{4k}(x_4) = \left\{\frac{\mu_2}{\mu_3}\left[M_1 - \left(M_1 \operatorname{tg} \frac{p_k}{a_2} l_2 + M_2\right) \operatorname{tg} \frac{p_k}{a_2} l_3 - M_2 \operatorname{tg} \frac{p_k}{a_2} l_2\right] \sin \frac{p_k}{a_3} x_4 + \left[M_1 \operatorname{tg} \frac{p_k}{a_2} l_2 + M_2 + \right.\right. \\
&\left.\left. + \left(M_1 - M_2 \operatorname{tg} \frac{p_k}{a_2} l_2\right) \operatorname{tg} \frac{p_k}{a_2} l_3\right] \cos \frac{p_k}{a_3} x_4\right\} \cos \frac{p_k}{a_1} l_1 \cos \frac{p_k}{a_2} l_2 \cos \frac{p_k}{a_2} l_3.
\end{aligned} \tag{29}$$

Based on the discovered natural functions (29), solutions $w_i(x_i, t)$ are written as series:

$$w_i(x_i, t) = \sum_{k=1}^{\infty} X_{ik}(x_i) \cdot T_{ik}(t), \quad i = 1-4. \tag{30}$$

Next, let us determine the time function $T_{ik}(t)$, i = 1 – 4, so that the series (30) satisfy the system of equations (23) and the initial conditions (24). Let us decompose the right parts of differential equations system (23) and the initial conditions (24) at intervals (0, l_i) on their own functions (29) according to Styeklov's decomposition theorem:

$$\begin{aligned}
&q_i(x_i, t) = \sum_{k=1}^{\infty} C_{ik}(t) X_{ik}(x_i); \\
&w_i(x_i, 0) = \sum_{k=1}^{\infty} T_{ik}(0) X_{ik}(x_i); \\
&\dot{w}_i(x_i, 0) = \sum_{k=1}^{\infty} \dot{T}_{ik}(0) X_{ik}(x_i),
\end{aligned} \tag{31}$$

where:

$$C_{ik}(t) = \int_0^{l_i} q_i(x_i, t) X_{ik}(x_i) dx_i \Big/ \int_0^{l_i} X_{ik}^2(x_i) dx_i;$$

$$T_{ik}(0) = \int_0^{l_i} w_{ik}(x_i, 0) X_{ik}(x_i) dx_i \Big/ \int_0^{l_i} X_{ik}^2(x_i) dx_i;$$

$$\dot{T}_{ik}(0) = \int_0^{l_i} \dot{w}_{ik}(x_i, 0) X_{ik}(x_i) dx_i \Big/ \int_0^{l_i} X_{ik}^2(x_i) dx_i.$$

According to (24) $\dot{w}_1(x_1, 0) = \dot{w}_2(x_2, 0) = 0$, then $\dot{T}_{1k}(0) = \dot{T}_{2k}(0) = 0$, $k = 1, \ldots, \infty$.

As a result, the system of inhomogeneous dynamic equations allowed to obtain a system of an infinite number of time function equations which are a classical Cauchy problem:

$$\begin{aligned}
&\ddot{T}_{ik}(t) + 2h_i \dot{T}_{ik}(t) + p_k^2 T_{ik}(t) = C_{ik}(t); \; T_{ik}(0); \\
&\dot{T}_{ik}(0), \; 0 \; i = \overline{1,4}, \; k = \overline{1,\infty}.
\end{aligned} \tag{32}$$

Then the time function takes the form:

$$\begin{aligned}
&T_{ik}(t) = e^{-h_i t}\left[B1_{ik} \sin\left(t\sqrt{p_k^2 - h_i^2}\right) + B2_{ik} \cos\left(t\sqrt{p_k^2 - h_i^2}\right)\right] + \\
&+ \frac{C_{ik}(t)}{\sqrt{\left(p_k^2 - \omega^2\right)^2 + 4h_i^2 \omega^2}},
\end{aligned} \tag{33}$$

and stable integration:

$$\begin{aligned}
&B1_{ik} = \frac{1}{\sqrt{p_k^2 - h_i^2}}\left[h_i T_{ik}(0) + \dot{T}_{ik}(0) + \frac{\dot{C}_{ik}(0) - h_i C_{ik}(0)}{\sqrt{\left(p_k^2 - \omega^2\right)^2 + 4h_i^2 \omega^2}}\right]; \\
&B2_{ik} = T_{ik}(0) - \frac{C_{ik}(0)}{\sqrt{\left(p_k^2 - \omega^2\right)^2 + 4h_i^2 \omega^2}}.
\end{aligned} \tag{34}$$

Thus, the solution of equation (1) for the third and fourth sections of the drill string is:

$$
\begin{aligned}
&u_3(x_3,t)=(m_0+a_{1S}-\mu m_5)\frac{g}{c_0}+g\sum_{j=1}^{2}\left(a_{jS}-\mu m_5-\frac{m_j}{2}\right)\frac{l_j}{E_jF_j}+(a_{3S}-\mu m_5)\frac{gx_3}{E_2F_2}-\frac{g\rho_2}{2E_2}x_3^2-\frac{x_3(l_3-x_3)}{E_2F_2l_3}P(t)+\\
&\sum_{k=1}^{\infty}\left[M_1\sin\frac{p_k}{a_2}(x_3+l_2)+M_2\cos\frac{p_k}{a_2}(x_3+l_2)\right]\left[e^{-h_3t}(B1_{3k}\sin p_{3k}t+B2_{3k}\cos p_{3k}t)+\frac{C_{3k}(t)}{\omega_{3k}}\right]\cos\frac{p_k}{a_1}l_1;\\
&u_4(x_4,t)=(m_0+a_{1S}-\mu m_5)\frac{g}{c_0}+g\sum_{j=1}^{2}\left(a_{jS}-\mu m_5-\frac{m_j}{2}\right)\frac{l_j}{E_jF_j}+\left(a_{3S}-\mu m_5-\frac{m_3}{2}\right)\frac{gl_3}{E_2F_2}+(a_{4S}-\mu m_5)\frac{gx_4}{E_3F_3}-\\
&-\frac{g\rho_3}{2E_3}x_4^2+\frac{x_4(l_4-x_4)}{E_3F_3l_4}P(t)+\sum_{k=1}^{\infty}\left\{\left[M_1-M_2\operatorname{tg}\frac{p_k}{a_2}l_2-\left(M_1\operatorname{tg}\frac{p_k}{a_2}l_2+M_2\right)\operatorname{tg}\frac{p_k}{a_2}l_3\right]\frac{\mu_2}{\mu_3}\sin\frac{p_k}{a_3}x_4+\right.\\
&\left.+\left[\left(M_1-M_2\operatorname{tg}\frac{p_k}{a_2}l_2\right)\operatorname{tg}\frac{p_k}{a_2}l_3+M_1\operatorname{tg}\frac{p_k}{a_2}l_2+M_2\right]\cos\frac{p_k}{a_3}x_4\right\}\cos\frac{p_k}{a_1}l_1\cos\frac{p_k}{a_2}l_2\cos\frac{p_k}{a_2}l_3\times\\
&\times\left[e^{-h_4t}(B1_{4k}\sin p_{4k}t+B2_{4k}\cos p_{4k}t)+\frac{C_{4k}(t)}{\omega_{4k}}\right].
\end{aligned}
\tag{35}
$$

So, moving of drill string cross-cuts (35) includes four components:

– stationary, which is determined by the parameters of the hoisting system, the drill string tension, pipes weight and drilling fluid density, viscous resistance power, and rock pressure on the DS stuck part . This component fully determines the location, length and strength of sticking;

– forced oscillations with the frequency of the exciting force and the amplitude, which depends on DS parameters, vibrator location in the heavy weight drill pipe and the amplitude of the disturbing force. Such fluctuations have the shape of standing waves and reach maximum amplitude the middle points of the sections. These waves do not transfer energy from the vibrator to the DS sticking place, but only convert kinetic energy into potential;

– inherent attenuating oscillations consisting of the sum of aliquant harmonics;

– forced oscillations with the frequency of the exciting force and the amplitude, which depends on DS parameters, vibrator location in the heavy weight drill pipe and natural DS frequencies, as well as the amplitude of the disturbing force. Such oscillations have the shape of traveling waves that spread energy in the DS. It is possible to enhance efficiency of energy transfer by tuning the vibrator to the resonance – one of the DS natural frequencies. (Bailey & Gupta, 2008).

4. MATERIALS UNDER ANALYSIS

Using mathematical modeling for the selected layout of the DS, we designed and built graphs for all components of drill pipes cross-cuts displacements (Kelly, 2008). Figure 2 shows elastic deformations of the drill pipe cross-cut above the DS sticking zone.

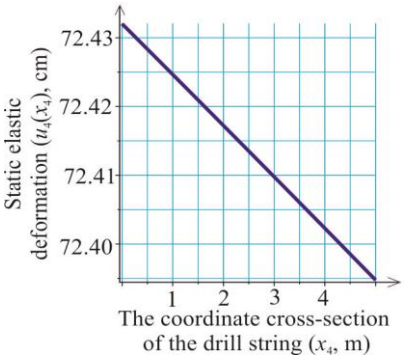

Figure 2. Static movement of the loose part of the drilling pipe

Figure 2 demonstrates that the extended cross-cut $u_4(0, t)$ begins to shrink as it approaches the accident zone $u_4(l_4, t) = u_4(0, t)$. Integral values of the sticking force and sticking length can be calculated from equations (15), taking into account the parameters of DS, hoisting system, and the physical properties of the drilling fluid and friction forces.

If the resistance is viscous, as in the chosen layout, simple oscillations attenuate exponentially (Fig. 3). The rate of decrease in the amplitude of these oscillations is determined by the logarithmic decrement of oscillations $\Lambda_{ik} = \alpha_i \pi / 2m_i p_k$, which is determined by the viscosity of drilling fluids. It should be noted that logarithmic decrement of the first section oscillations $\Delta_{1\max} = 0.0004$ is small compared to the logarithmic decrement of the fourth section oscillations $\Delta_{4\max} = 0.15$.

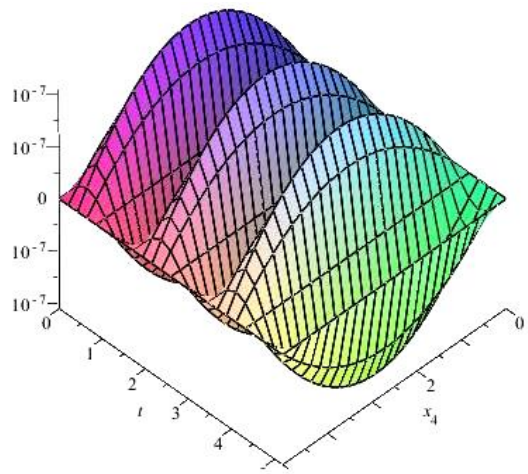

Figure 3. Standing waves when configuring the vibration device on the first frequency of the drill string

Forced oscillations of the standing waves for the selected DS layout were calculated for harmonic disturbance $P(t) = A\sin(\omega t + \gamma)$. with amplitude A = 1kN.

Figure 3 demonstrates standing waves when the vibrator is configured to the first natural DS frequency, and Figure 4 – to the fifth natural frequency. Thus, the frequency increases with decreasing oscillation period.

For the studied drill string, the oscillations of standing waves were low amplitude, and therefore their influence $\varphi_i(x_i, t)$ on the drill string vibration $u_i(x_i, t)$ can be neglected.

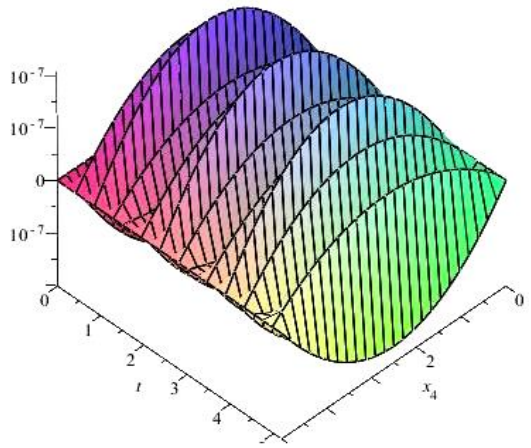

Figure 4. Standing waves when configuring the vibration device to the fifth frequency of the drill string

Figure 5 shows free oscillations of the first DS section that slowly fade away as they approach the borehole being close to a harmonious in character.

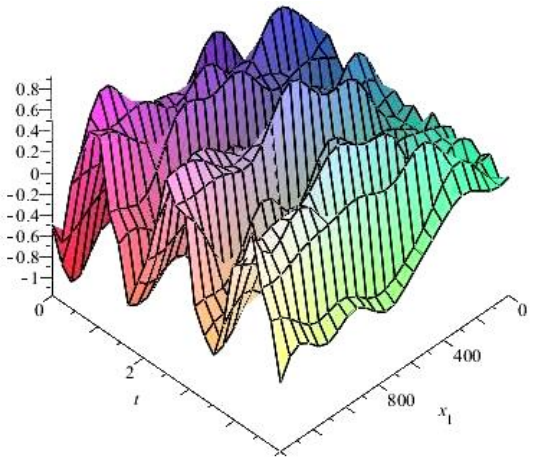

Figure 5. Simple oscillations of the first drill string section

Figure 6 and 7 show oscillations of HWDP that are beginning to slowly attenuate, and Figure 8 presents oscillations of the stuck drill pipe that attenuate rather quickly.

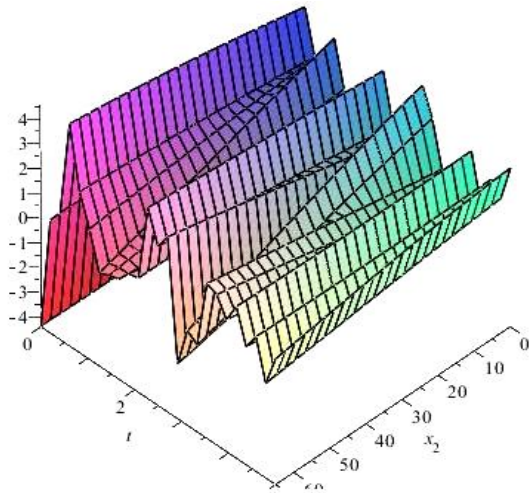

Figure 6. Simple oscillations of the heavy weight drill pipe above the vibrator installation location

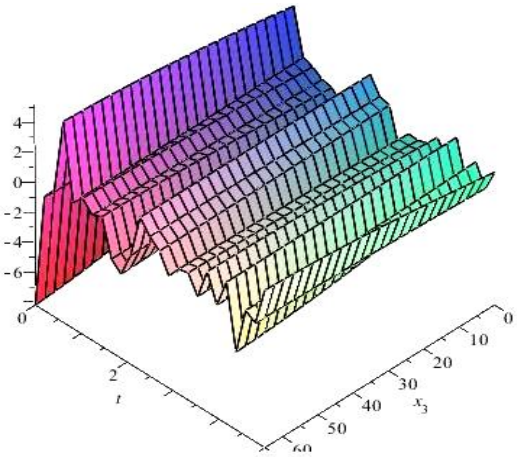

Figure 7. Simple oscillations of the heavy weight drill pipe below the vibrator location

The study into free oscillations has shown that they quickly fade in the course of approaching the accident zone.

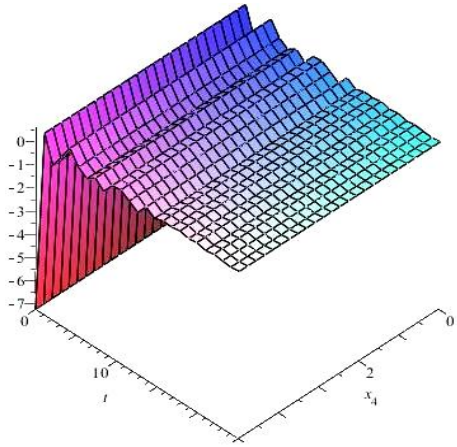

Figure 8. Simple oscillations of the drill string stuck section

Forced oscillations of the heavy weight drill pipe (Fig. 9) are multi-harmonious, but the intensity of the stuck DS section oscillations decreases sharply with the increase in resonance frequency.

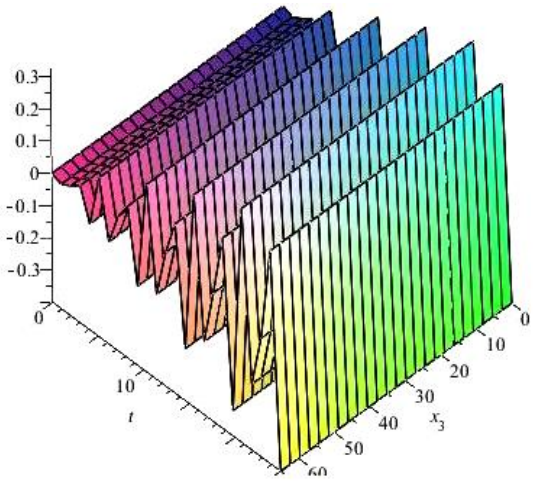

Figure 9. Forced oscillations of the heavy weight drill pipe

Figure 10 shows resonance achieved by configuring the vibrator to the first natural DS frequency, and Figure 11 – to the fifth natural frequency.

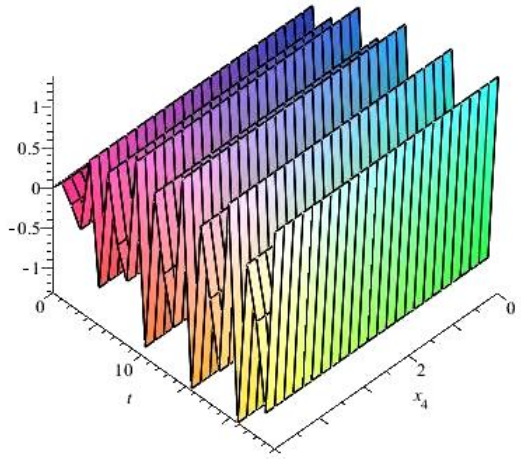

Figure 10. Forced oscillations of the stuck section when configuring the vibration device ton the first frequency of the drill string

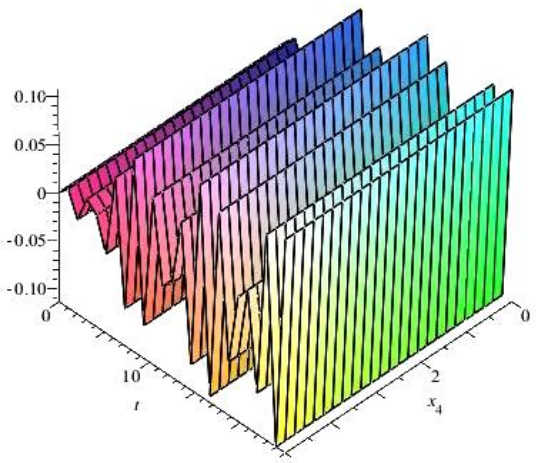

Figure 11. Forced oscillations of the stuck section when configuring the vibration device to the fifth frequency of the drill string

To analyze the setting up of the vibration mechanism to natural DS frequency, we calculated the resonance amplitudes (Table 2) of the forced oscillations in the sticking place and constructed the amplitude-frequency characteristic (Fig. 12).

Table 2. Resonance amplitudes of oscillations in the sticking zone

Resonance frequency	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
Amplitude, cm	144.6	40.8	22.3	15.2	11.6	9.3	8.0	7.1	6.4	5.7

Past studies have shown that the intensity of forced oscillations increases with the approach of the installation site vibrator in HWDP to the accident.

To analyze the vibration mechanism setting up on natural frequency DS calculated the resonance amplitude (Table 2) forced oscillations in the place and constructed stuck frequency response (Fig. 12).

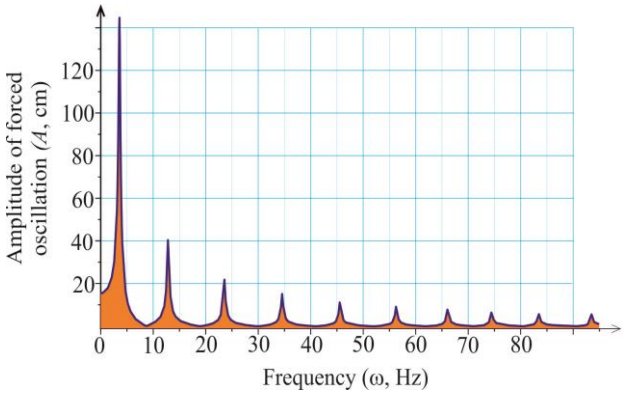

Figure 12. Amplitude-frequency characteristics of the drill string oscillations excited by the vibrator

Previous studies have shown that the intensity of forced oscillations increases as the vibrator installation place in HWDP approaches the accident zone (Table 3).

It is worth noting that along with the trend described above, coefficient of the forced oscillations amplitude transmission increases if the vibrator installation location is moved in HWDP in the direction of the borehole.

An important dynamic condition of drilling tool effective operation is instantaneous longitudinal forces that arise in cross-cuts of drill pipes:

$$P_i\left(x_i,t\right)=E_iF_iu_i'\left(x_i,t\right),\ i=\overline{1,4}. \quad (36)$$

Another important dynamic operational characteristics is strength testing of all cross-cuts of DS.

$$\sigma_i\left(x_i,t\right)=E_iu_i'\left(x_i,t\right),\ i=\overline{1,4}. \quad (37)$$

Research into the force variations in drill pipes cross-cuts showed that the vibrator installation locations $P_3\left(0,t\right)$ and $\sigma_3\left(0,t\right)$ are subjected to the greatest loads and stresses excited by external vibration.

Amplitude values of these forces and stresses (Fig. 13) for the selected DS layout grow when the vibrator is set up to a higher natural DS frequency and sharply fall when the vibrator is lowered along HWDP.

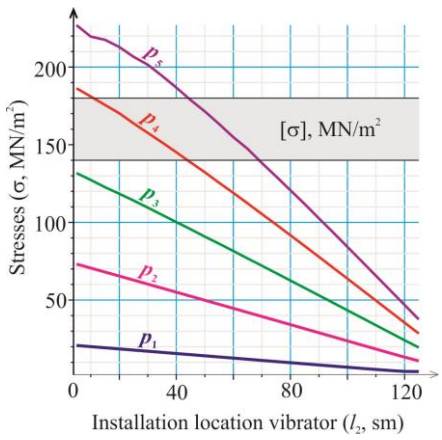

Figure 13. Amplitudes of forces and stresses in the vibrator installation location while its configuring to the natural DS frequencies p_k, k = 1.5

Table 3. The oscillations resonant amplitudes within the zone of vibrator location

Location the installation, m	The resonant amplitude, cm									
	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
5	144.1	38.9	18.8	9.9	4.4	6	1.7	2.5	2.2	1.0
10	144.1	39.0	19.0	10.3	4.9	1.2	1.2	2.2	2.3	1.4
15	144.2	39.2	19.2	10.7	5.4	1.7	0.7	2.0	2.4	1.7
20	144.2	39.3	19.5	11.1	5.9	2.3	0.2	1.6	2.2	1.9
25	144.2	39.4	19.8	11.4	6.4	2.9	0.4	1.1	2.0	2.1
30	144.3	39.5	20.0	11.7	6.9	3.4	1.0	0.6	1.7	2.1
35	144.3	39.7	20.2	12.1	7.3	4.0	1.6	0.5	1.3	2.0
40	144.3	39.8	20.4	12.4	7.7	4.5	2.2	0.6	0.8	1.7
45	144.4	39.9	20.6	12.6	8.1	5.0	2.8	1.1	0.2	1.4
50	144.4	40.0	20.7	12.9	8.5	5.4	3.3	1.8	0.4	0.9
55	144.4	40.1	20.9	13.2	8.8	5.9	3.8	2.3	1.0	0.3
60	144.4	40.1	21.1	13.4	9.1	6.3	4.3	2.9	1.6	0.3
65	144.4	40.2	21.2	13.6	9.5	6.7	4.8	3.4	2.2	0.9
70	144.4	40.3	21.4	13.8	9.7	7.0	5.2	3.9	2.7	1.5
75	144.5	40.4	21.5	14.0	10.0	7.4	5.6	4.4	3.3	2.1
80	144.5	40.4	21.6	14.2	10.2	7.7	6.0	4.8	3.7	2.7
85	144.5	40.5	21.7	14.4	10.5	8.0	6.3	5.2	4.2	3.2
90	144.5	40.5	21.8	14.5	10.6	8.2	6.6	5.5	4.6	3.6
95	144.6	40.6	21.9	14.7	10.8	8.4	6.9	5.9	4.9	4.1
100	144.6	40.6	22.0	14.8	11.0	8.7	7.1	6.1	5.3	4.4
105	144.6	40.7	22.1	14.9	11.1	8.8	7.4	6.4	5.5	4.7
110	144.6	40.7	22.1	15.0	11.2	9.0	7.5	6.6	5.8	5.0
115	144.6	40.7	22.2	15.0	11.3	9.1	7.7	6.7	6.0	5.2
120	144.6	40.8	22.2	15.1	11.4	9.2	7.8	6.9	6.1	5.4
125	144.6	40.8	22.3	15.2	11.5	9.3	7.9	7.0	6.2	5.5

For effective elimination of sticking it was necessary to study DS kinetic and potential energy as well as its dependence upon the location of the vibrator:

$$E_{kin}(t)=\frac{1}{2}\sum_{i=1}^{4}\rho_i F_i \int_0^{l_i} \dot{u}_i^2(x_i,t)dx_i;$$

$$E_{pot}(t)=\sum_{i=1}^{4}E_i F_i \int_0^{l_i} u_i'(x_i,t)dx_i,\ i=\overline{1,4}, \qquad (38)$$

where:

$\dot{u}_i(x_i,t)$ – speed of the current cross-cut of DS i-th section with coordinate x_i at an arbitrary time t.

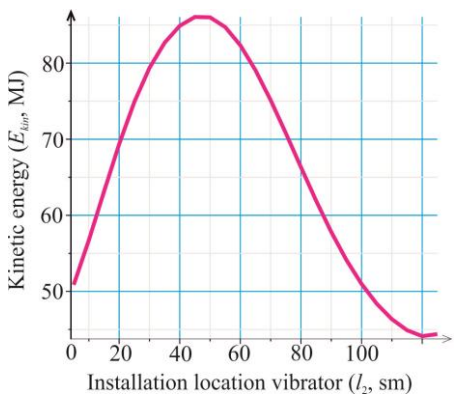

Figure 14. Energy characteristics of DS oscillations excited by the vibrator on the first three resonant frequencies

For the selected layout of the drill string, kinetic energy of the stuck DS is 1.34 MJ and potential – 2.18 MJ. Studies have shown that energy curves have clearly defined extremes (Fig. 14): the largest kinetic energy is accumulated by DS in case of placing the vibrator at a distance that is 0.36 HWDP length (Fig. 14), and maximum of potential energy – in the middle of HWDP. Thus kinetic energy constitutes only 5 – 10% of the potential energy (Fig. 15).

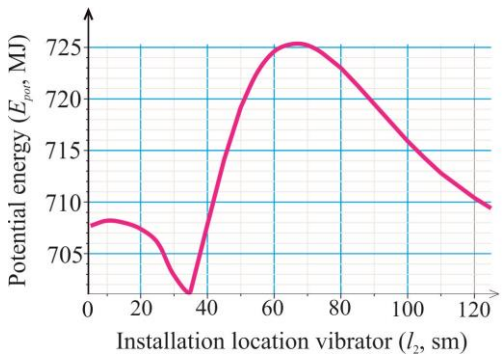

Figure 15. Amplitudes of potential energy

So, it is more feasible to eliminate accidents at the expense of DS elastic deformations that can be reinforced by the vibration device.

5. CONCLUSIONS

In this work we presented a discrete-continuum mathematical model of a drill string with vibration mechanism built into the HWDP. Based on the elaborated mathematical model we have developed a computer program which helped us to visualize oscillatory processes of the drill-string, as well as to carry out numerical calculations of power and energy performance.

The obtained results were analyzed. The developed model allows to promptly analyze and explain the choice of vibration mechanism location, so as to provide fast retrieving of the stuck drill string. Departing from the aforementioned results, we elaborated the following recommendations:

– the vibrator should be set up to one of its three natural frequencies;

– vibration device has to be situated at the distance which is 0.35 – 0.5 of the HWDP length, starting from its top. It will allow saving the biggest amount of potential energy – elastic deformation of the bottom of the drill-string loose part;

– in the case of setting up the vibration device to its natural frequencies (higher than the third), the vibration mechanism has to be located in HWDP as low as possible. This is because excited vibrations can cause destruction of drilling pipes;

– for every given layout of the drill-string we should carry out numerical calculations that will allow to predict any possible sticking, and also to choose location for the vibration mechanism;

– we can use the given results for further research and perfection of existing engineering methods of modeling and calculation of drill-strings on the stage of their designing and construction.

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ABSTRACT (IN UKRAINIAN)

Мета. Дослідити залежності динамічних характеристик коливальних рухів у місці застрявання бурильної колони від параметрів вібраційного механізму та місця його установки в обтяженій бурильній трубі. Окремо розглянути умови збурення резонансних коливань у бурильній колоні. За результатами проведених досліджень розробити методику застосування вібратора, рекомендації з вибору місця його встановлення в обтяженій бурильній трубі, настроювання на резонансну частоту, вибору амплітуди збурювальної сили.

Методика. Для проведення експериментальних досліджень отримана дискретно-континуальна модель, в якій враховано хвильові процеси. Запропоновано математичну модель роботи бурильної колони з вібраційним механізмом, вмонтованим в обтяжену бурильну трубу.

Результати. На основі математичної моделі складено комп'ютерну програму для візуалізації коливальних процесів, що відбуваються в колоні труб, та чисельного розрахунку основних кінематичних і динамічних характеристик досліджуваної системи. Розраховано частотний спектр власних коливань для вибраної компоновки бурильної колони, а також резонансні амплітуди на цих частотах.

Наукова новизна. Запропонована методика дозволяє забезпечити суттєве підвищення точності визначення зусиль, напружень і запасів міцності у довільному перерізі колони труб, а також прогнозувати прихоплення бурильних колон при бурінні нафтових і газових свердловин.

Практична значимість. Наведено методику вибору місця монтування вібратора. Розроблено рекомендації з вибору місця установки, амплітуди збуреної сили й резонансних частот для ліквідації прихоплень труб и попередження руйнування бурильних колон.

Ключові слова: *буріння, бурильна колона, пружні хвилі, математична модель, вібрація, прихоплення, вібраційний пристрій, напруження, енергія прихопленої колони*

ABSTRACT (IN RUSSIAN)

Цель. Исследовать зависимости динамических характеристик колебательных движений в месте прихвата бурильной колонны от параметров вибрационного механизма и места его установки в утяжеленной бурильной трубе. Отдельно рассмотреть условия возбуждения резонансных колебаний в бурильной колоне. По результатам проведенных исследований разработать методику использования вибратора, рекомендации по выбору места его установки в утяжеленной бурильной трубе, настраивания на резонансную частоту, выбора амплитуды вынужденной силы.

Методика. Для проведения экспериментальных исследований получена дискретно-континуальная модель, в которой учтены волновые процессы. Предложенная математическая модель работы бурильной колонны с вибрационным механизмом, вмонтированным в утяжеленную бурильную трубу. Вибратор используется для ликвидации прихватов бурильного инструмента.

Результаты. На основании этой модели составлена компьютерная программа с целью визуализации колебательных процессов, происходящих в колонне труб, и числового расчета основных кинематических и динамических характеристик исследуемой системы. Произведен расчет частотного спектра собственных колебаний для выбранной компоновки бурильной колонны, а также резонансных амплитуд на этих частотах.

Научная новизна. Предложенная методика позволяет обеспечить существенное повышение точности определения усилий, напряжений и запасов прочности в произвольном сечении колонны труб и прогнозировать прихваты бурильных колонн при бурении нефтяных и газовых скважин.

Практическая значимость. Приведена методика выбора места монтирования вибратора. Даны рекомендации по выбору места установки, амплитуды возбуждающей силы и резонансных частот для ликвидации прихвата труб и предупреждения разрушения бурильных колонн.

Ключевые слова: *бурение, бурильная колонна, упругие волны, математическая модель, вибрация, прихват, вибрационное устройство, напряжение, энергия прихваченной колонны*

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